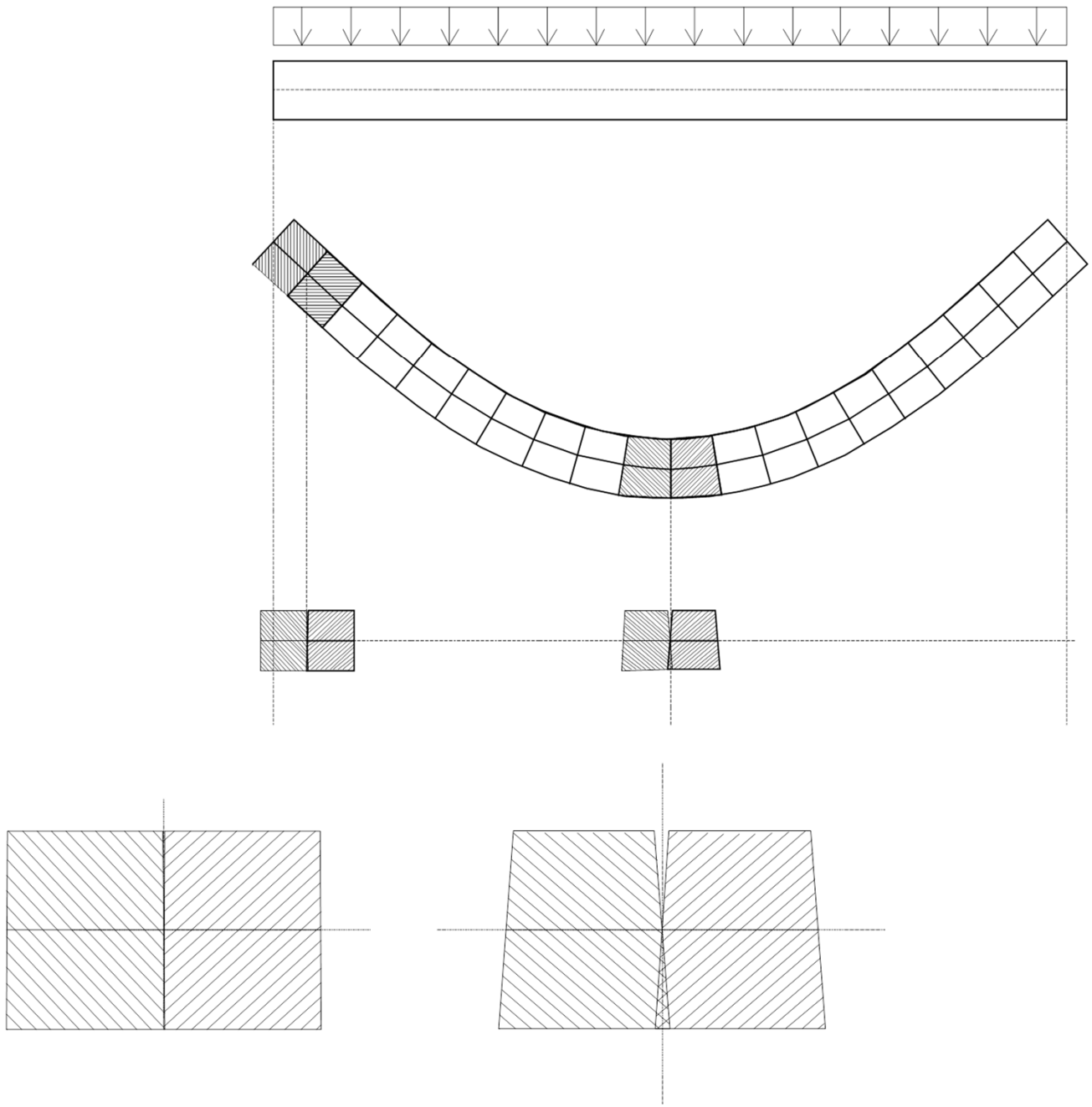
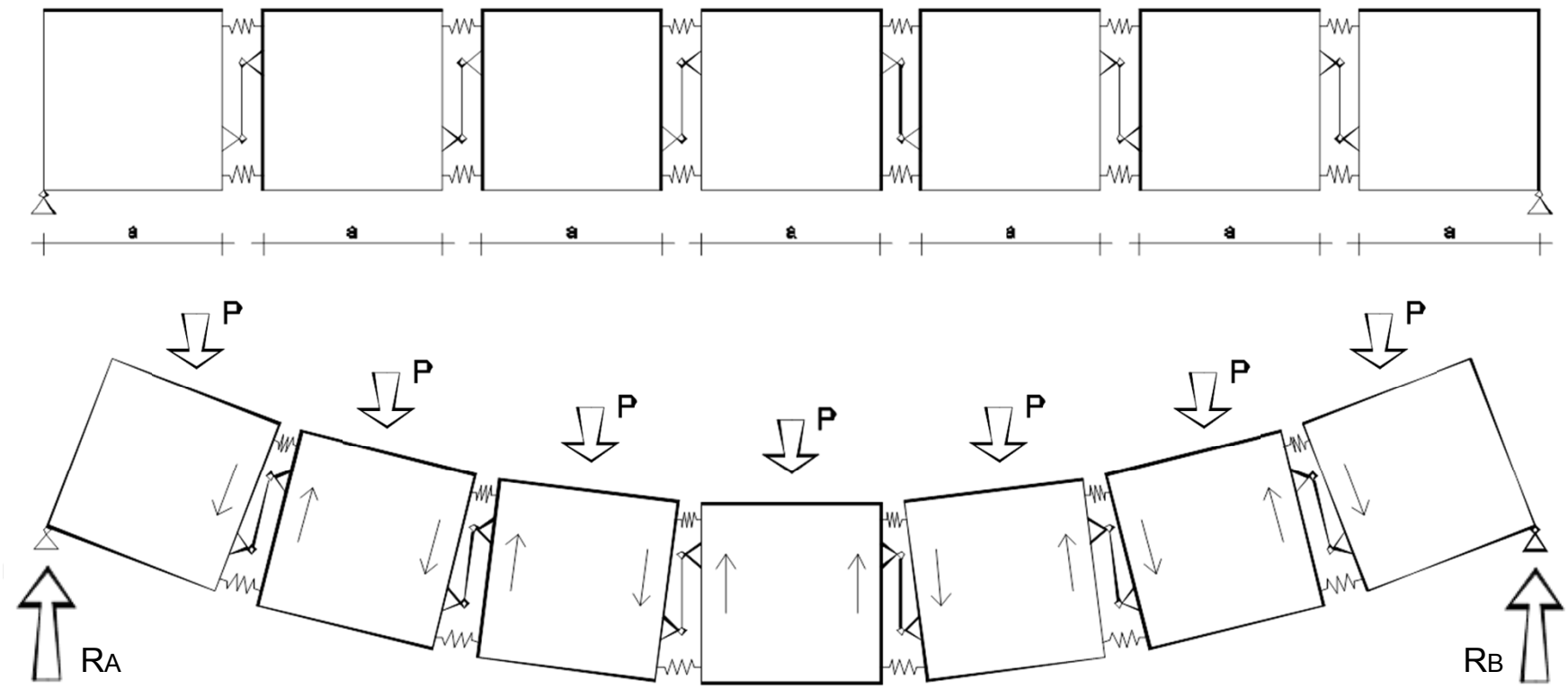
TERCERA FAMILIA de ESTRUCTURAS: ESTRUCTURAS FLEXADAS

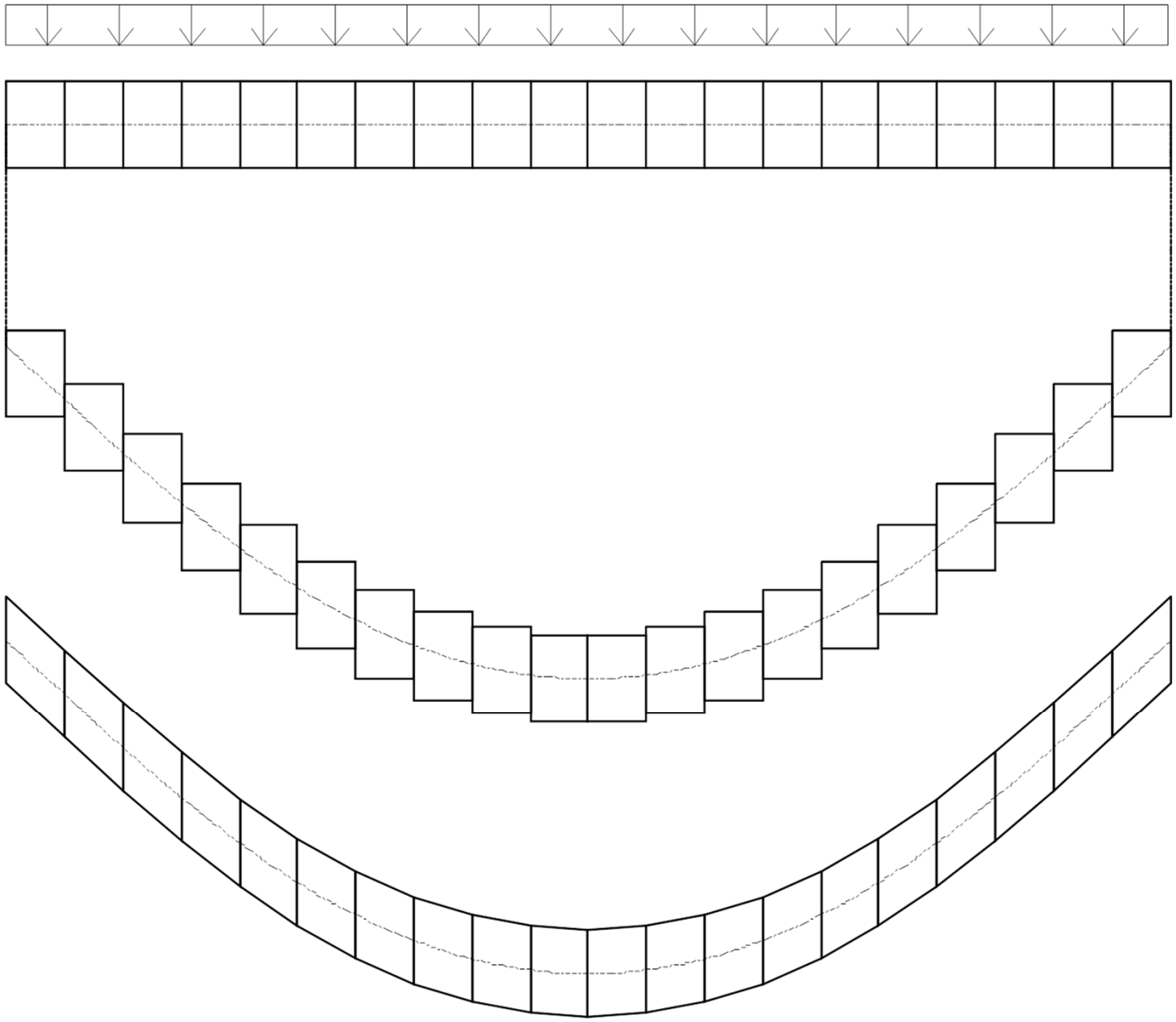
2. RELACIONES entre CARGA (p), CORTANTE (V) y MOMENTO (M).



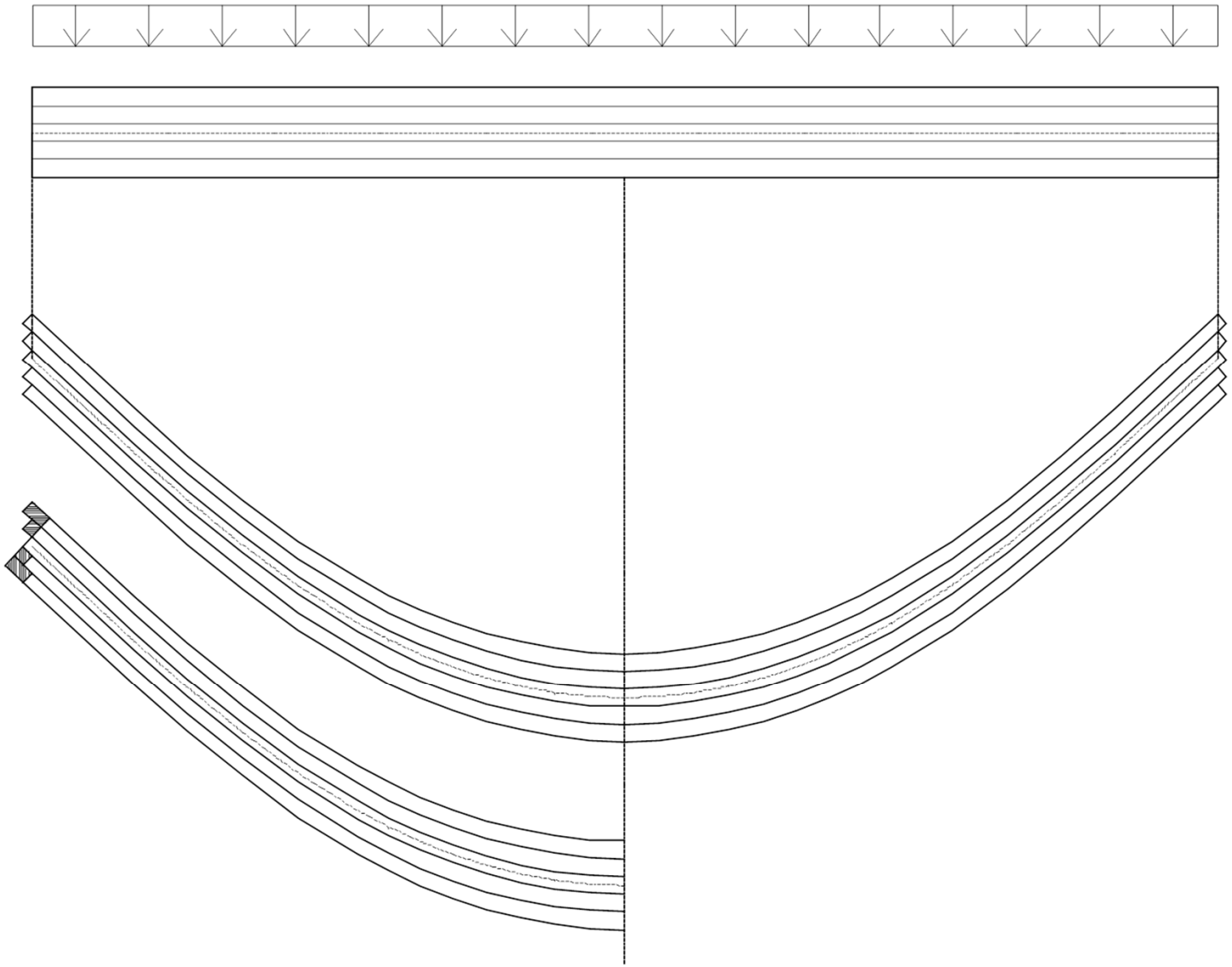
PRIMER MODELO DE LAS DOVELAS



SEGUNDO MODELO DE LAS DOVELAS

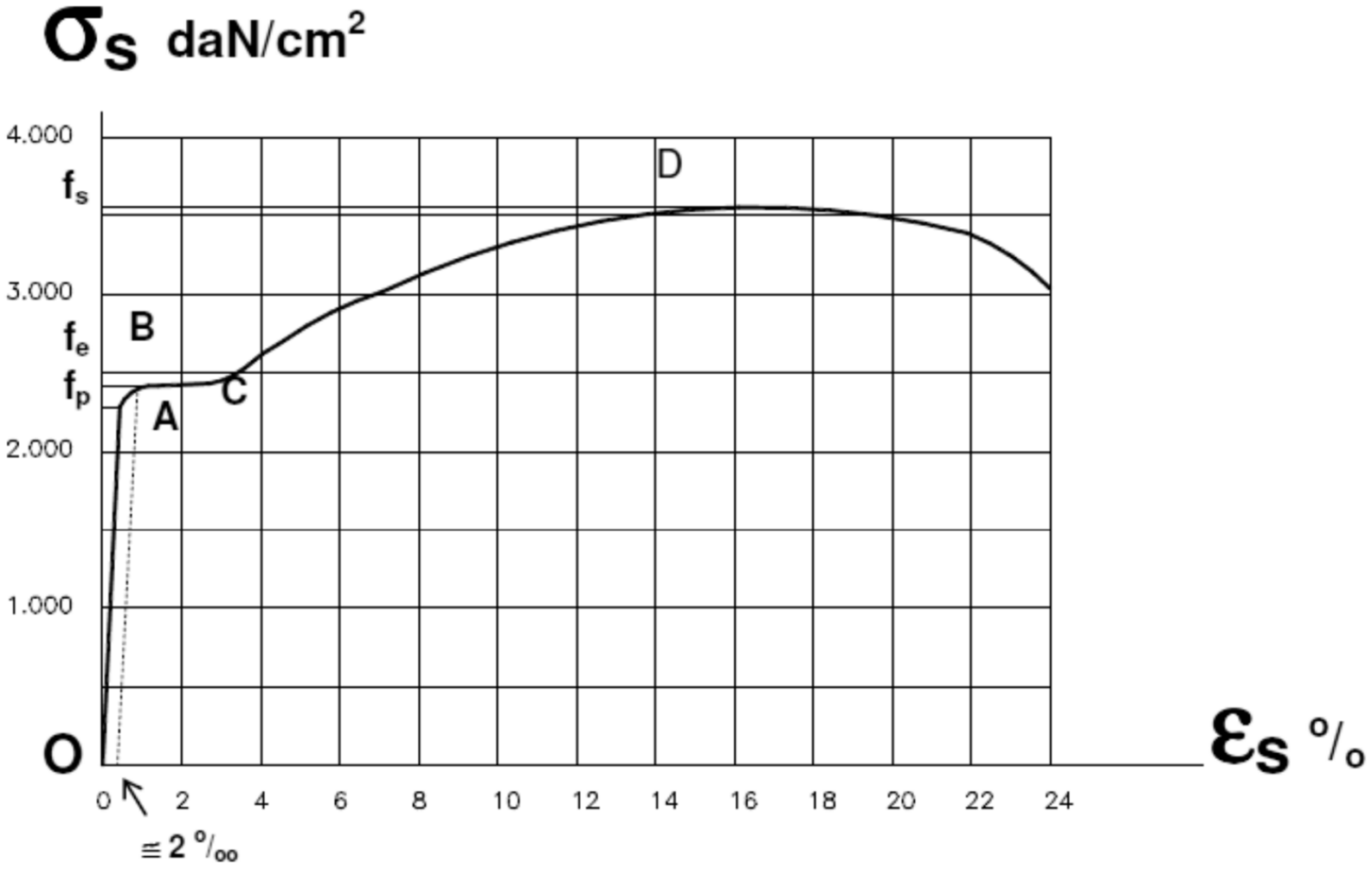


TERCER MODELO DE LAS DOVELAS

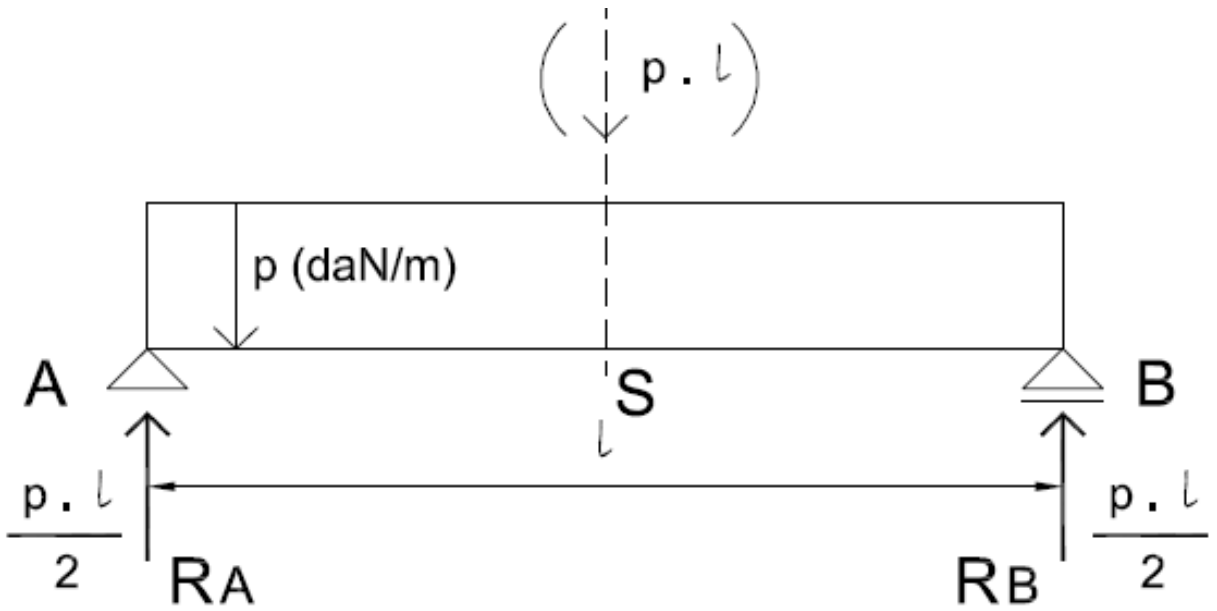


CUARTO MODELO DE LAS TABLILLAS

Gráfico de tensiones-deformaciones del acero



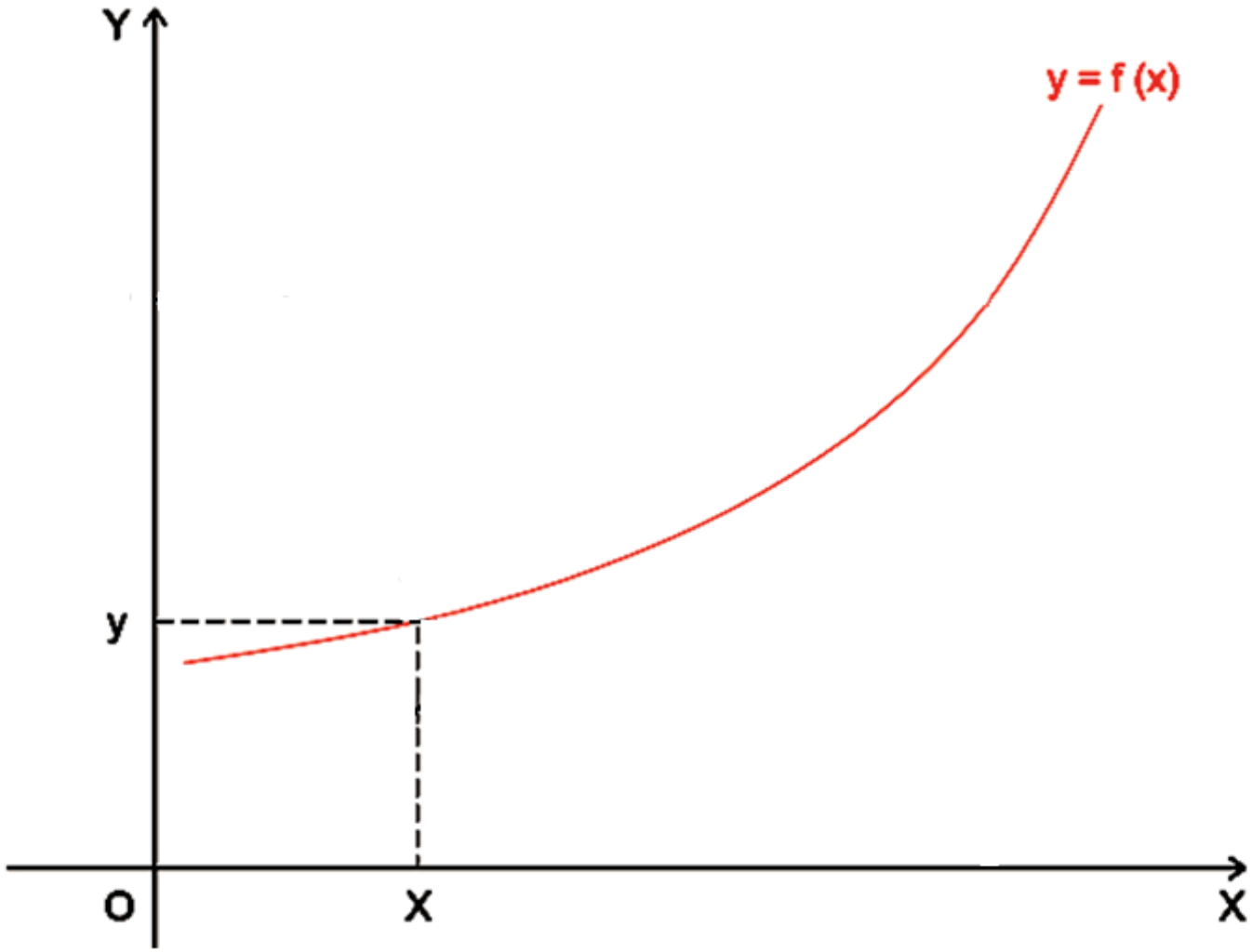
SOLICITACIONES EN UNA VIGA A FLEXIÓN SIMPLE



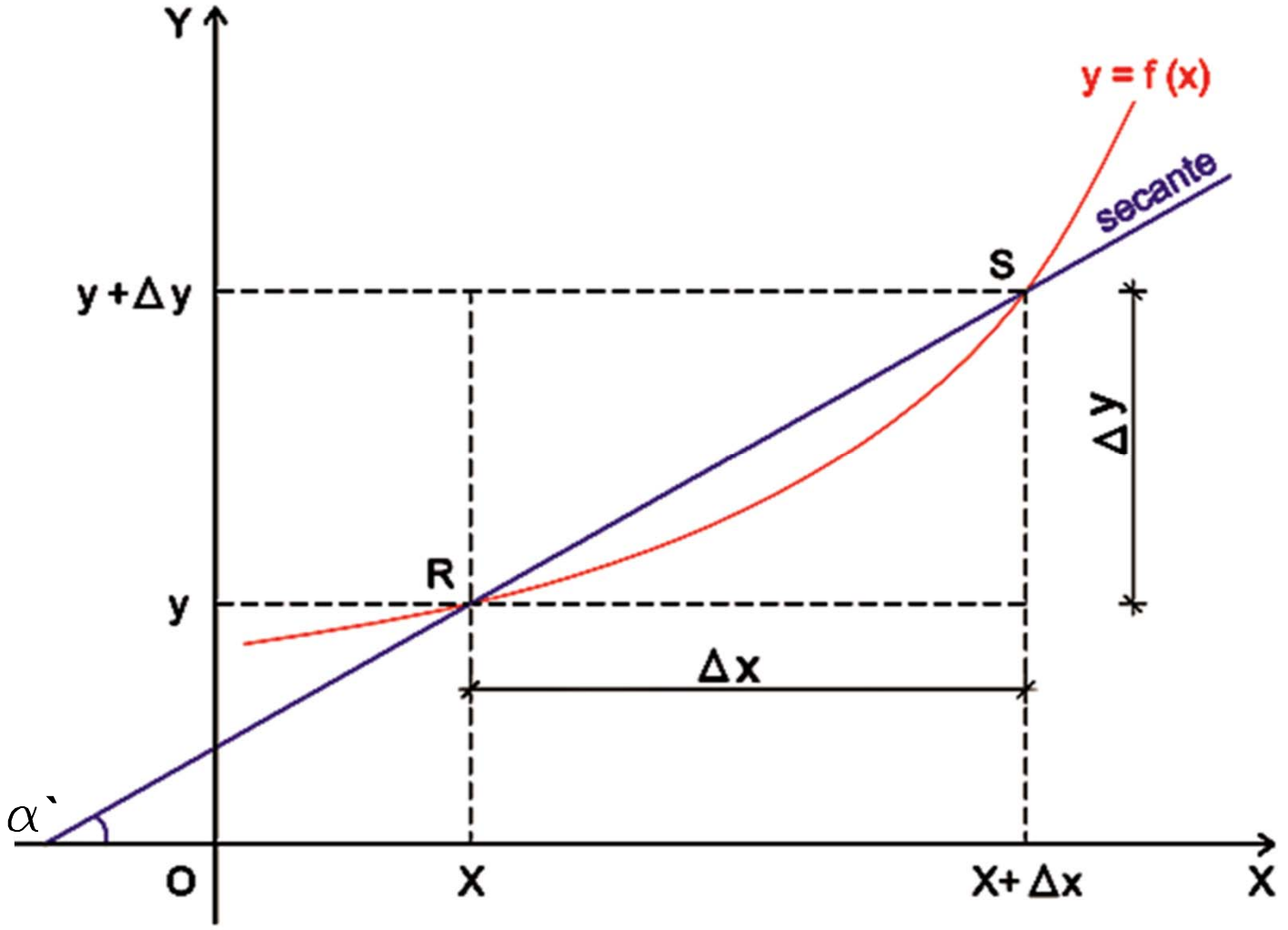
$$V_A = \frac{p \cdot l}{2} \qquad V_S = 0 \qquad V_B = -\frac{p \cdot l}{2}$$

$$M_A = 0 \qquad M_S = \frac{p \cdot l^2}{8} \qquad M_B = 0$$

FUNCIÓN



DERIVADA



$$tg\alpha' = \frac{\Delta y}{\Delta x}$$ (coeficiente angular o pendiente de la recta secante)

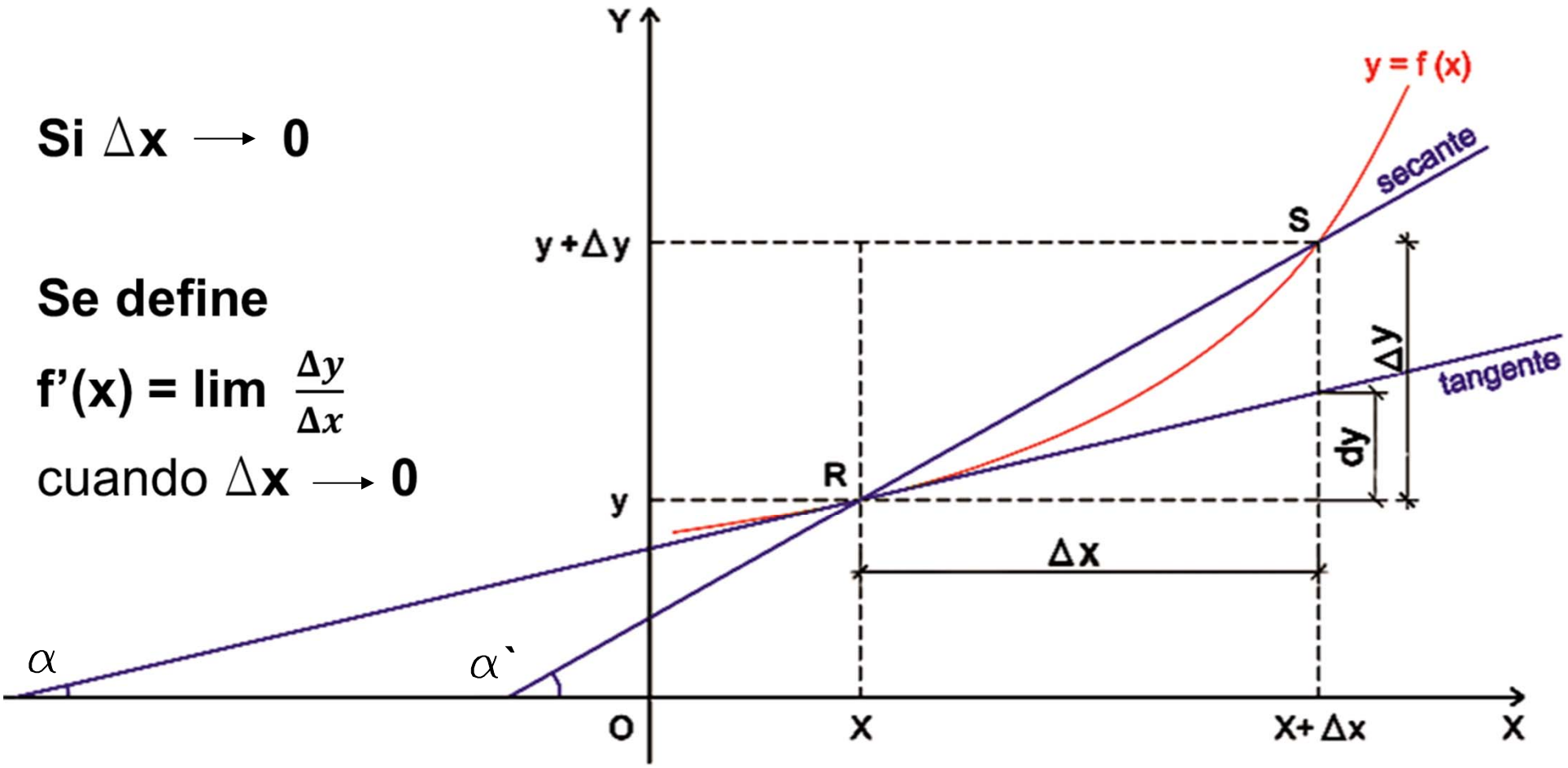
DERIVADA

Si $\Delta x \longrightarrow 0$

Se define

$f'(x) = \lim \frac{\Delta y}{\Delta x}$

cuando $\Delta x \longrightarrow 0$



La derivada en un punto representa el coeficiente angular o pendiente de la recta tangente a la función en ese punto.

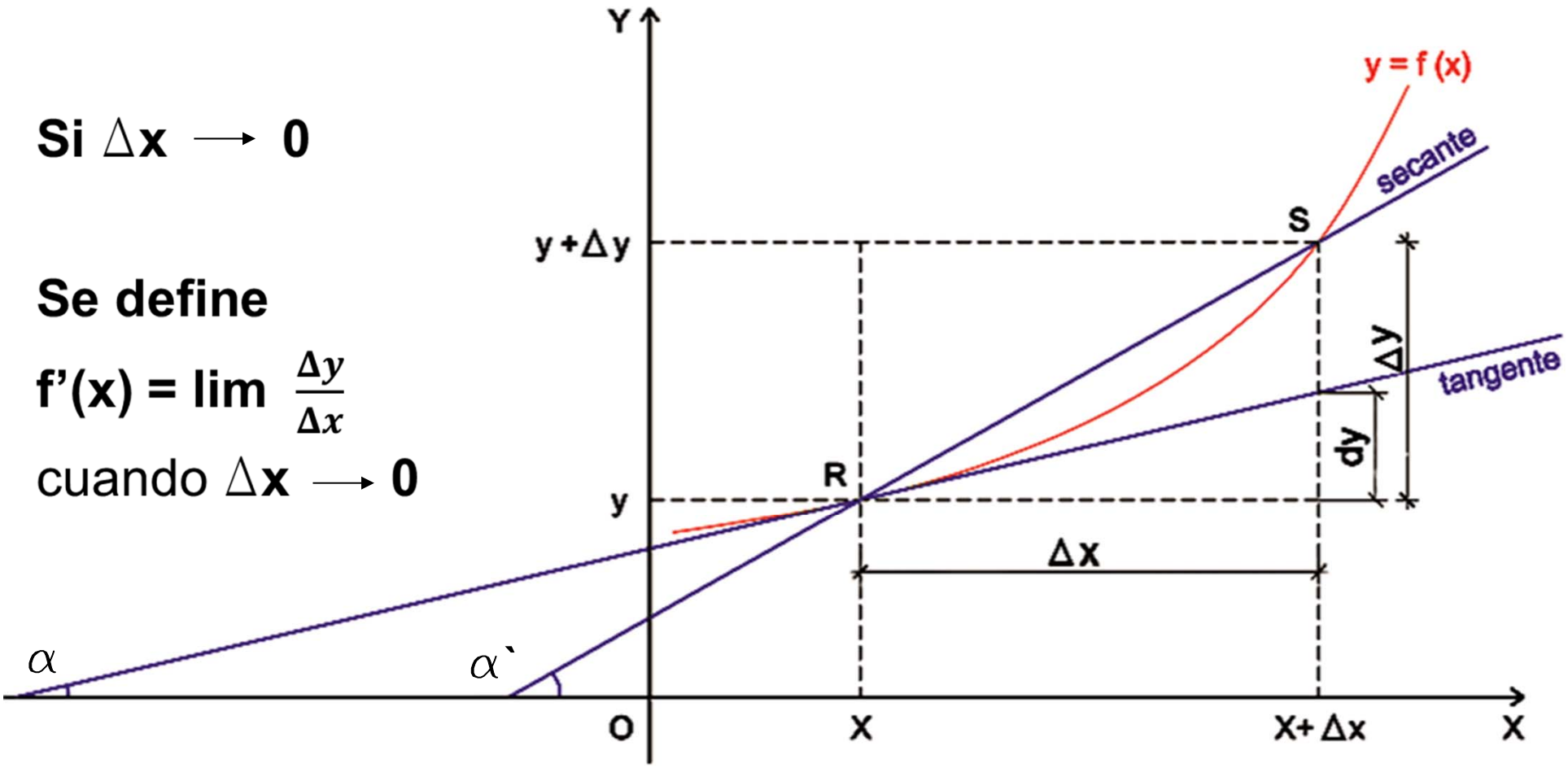
DERIVADA

Si $\Delta x \longrightarrow 0$

Se define

$f'(x) = \lim \frac{\Delta y}{\Delta x}$

cuando **$\Delta x \longrightarrow 0$**



$tg\ \alpha = f'(x) = \boxed{\frac{dy}{dx} = y'(x)}$

$dx \approx \Delta x$ **si $\Delta x \longrightarrow 0$** $\therefore$ **$\Delta y \approx dy$**

INTEGRAL INDEFINIDO

F (x) es primitiva de f (x) si F'(x) = f (x)

es la operación contraria a la derivación

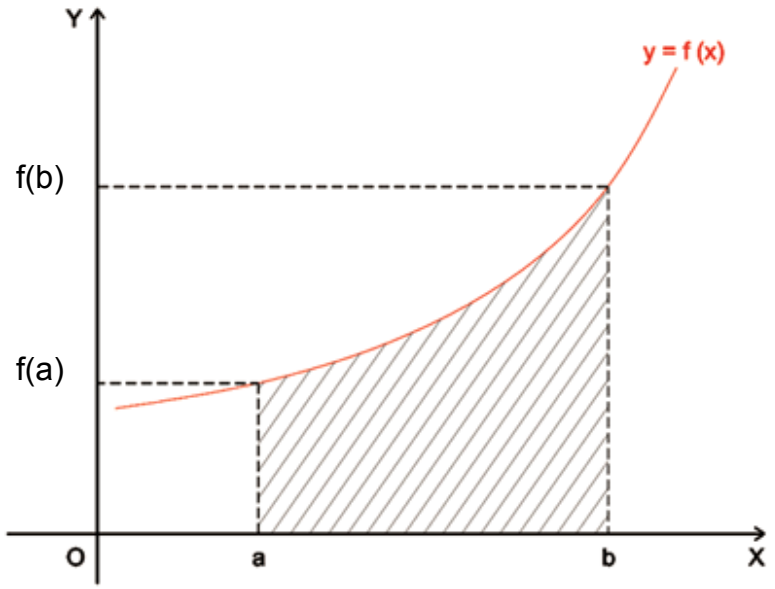
Ejemplo: $f(x) = x^2$ $\qquad$ $f(x) = x^n$

$f'(x) = 2 \cdot x$ $\qquad$ $f'(x) = n \cdot x^{n-1}$

$F(x) = \frac{x^3}{3} + c$ $\qquad$ $F(x) = \frac{x^{n+1}}{n+1} + c$

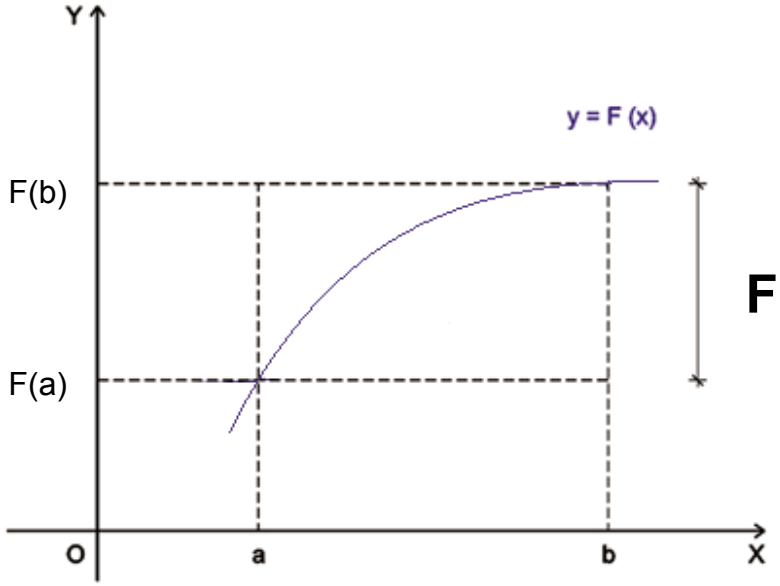
$$\int f(x)\,dx = F(x) + c$$

INTEGRAL DEFINIDO



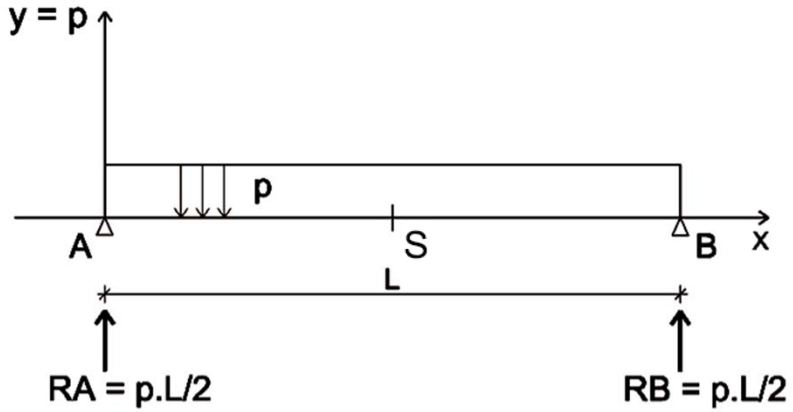
$$\int_a^b f(x)\,dx = F(b) - F(a)$$

siendo F(x) una primitiva de f(x)



F (b) – F(a)

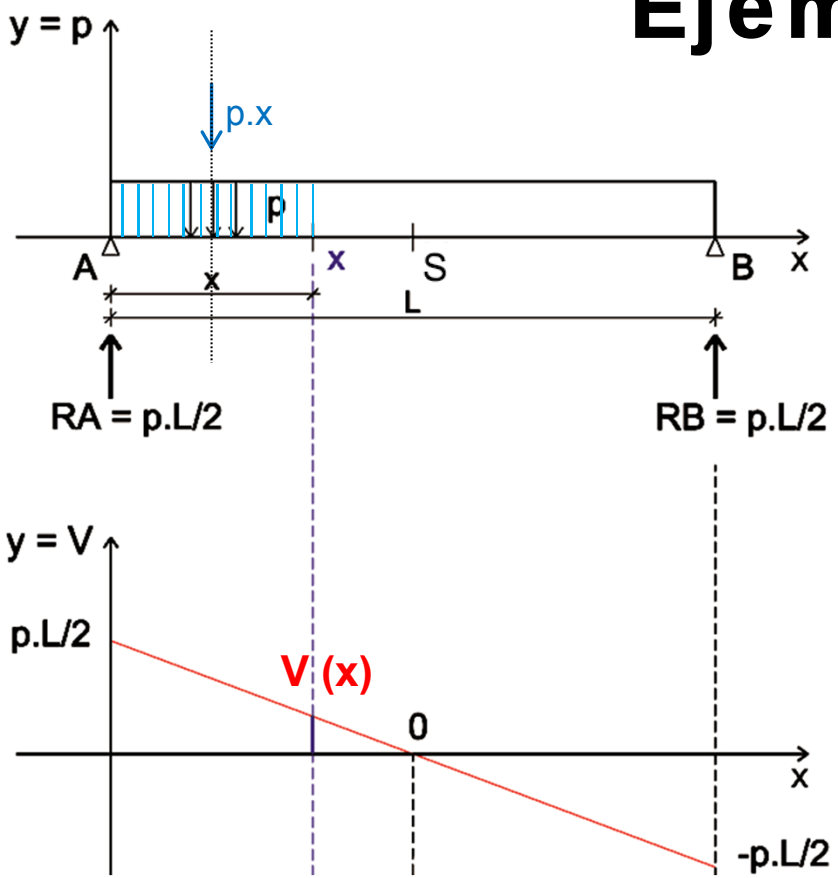
Ejemplo: viga



$$V_A = \frac{p \cdot l}{2} \qquad V_S = 0 \qquad V_B = -\frac{p \cdot l}{2}$$

$$M_A = 0 \qquad M_S = \frac{p \cdot l^2}{8} \qquad M_B = 0$$

Ejemplo: viga



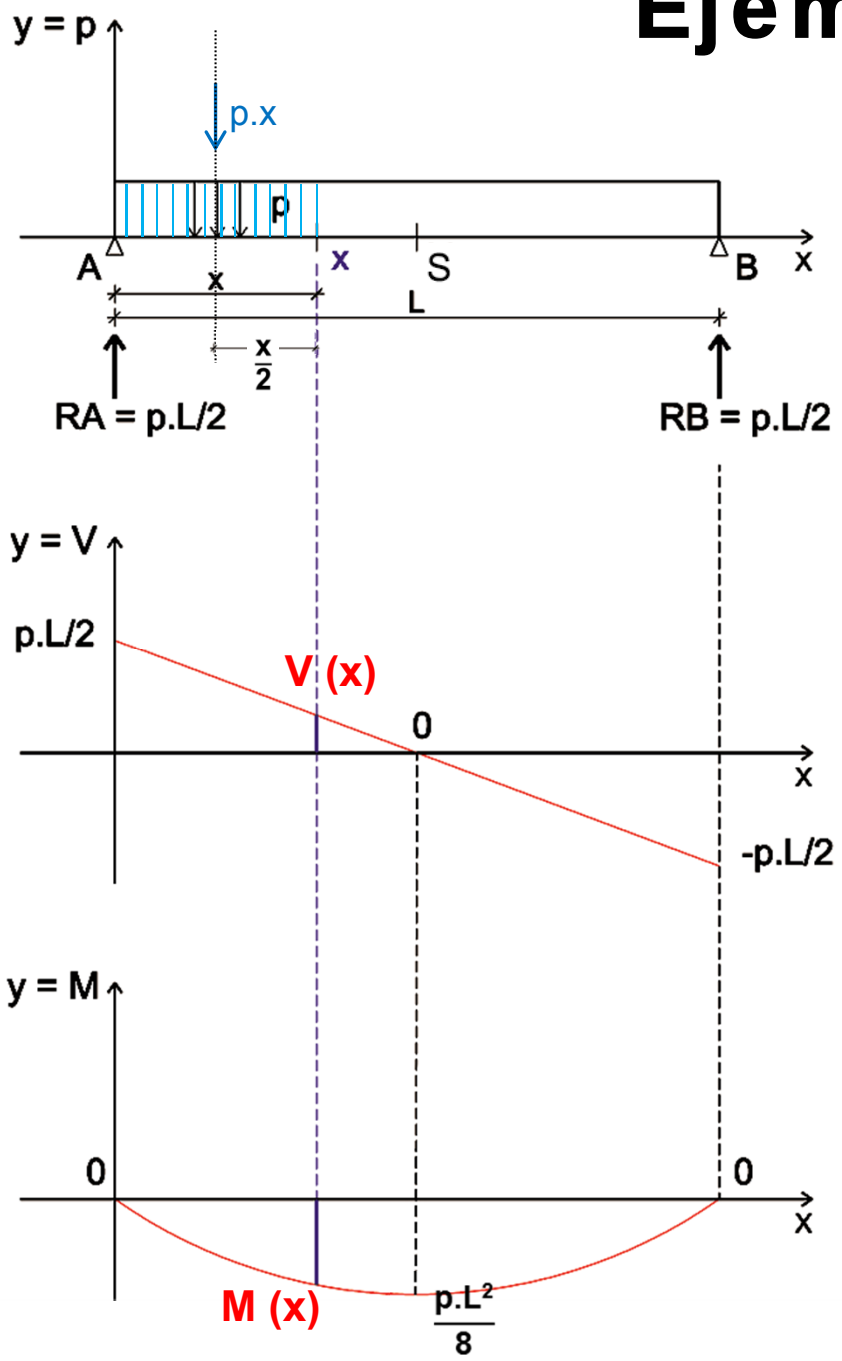
$$V(x) = \frac{p.L}{2} - p.x$$

$$V(0) = \frac{p.L}{2}$$

$$V(L/2) = 0$$

$$V(L) = -\frac{p.L}{2}$$

Ejemplo: viga



$$V(x) = \frac{p.L}{2} - p.x$$

$$V(0) = \frac{p.L}{2}$$

$$V(L/2) = 0$$

$$V(L) = -\frac{p.L}{2}$$

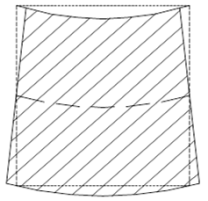
$$M(x) = \frac{p.L}{2}.x - p.x.\frac{x}{2}$$

$$M(x) = \frac{p.L}{2}.x - \frac{p.x^2}{2}$$

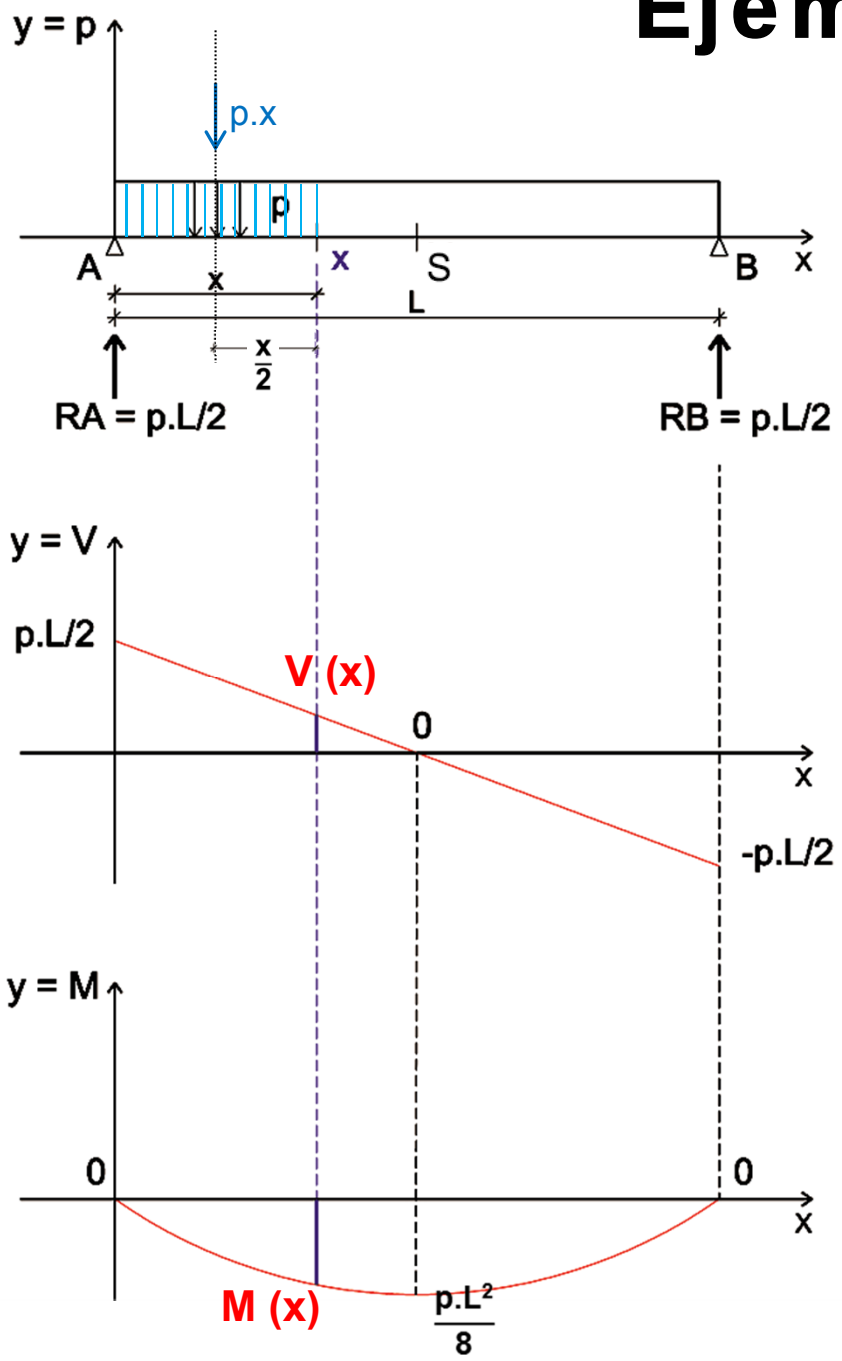
$$M(0) = 0$$

$$M(L/2) = \frac{p.L^2}{8}$$

$$M(L) = 0$$



Ejemplo: viga



$$M(x) = \frac{p.L}{2}.x - \frac{p.x^2}{2}$$

$$M`(x) = \frac{p.L}{2} - (2.\frac{p}{2}).x$$

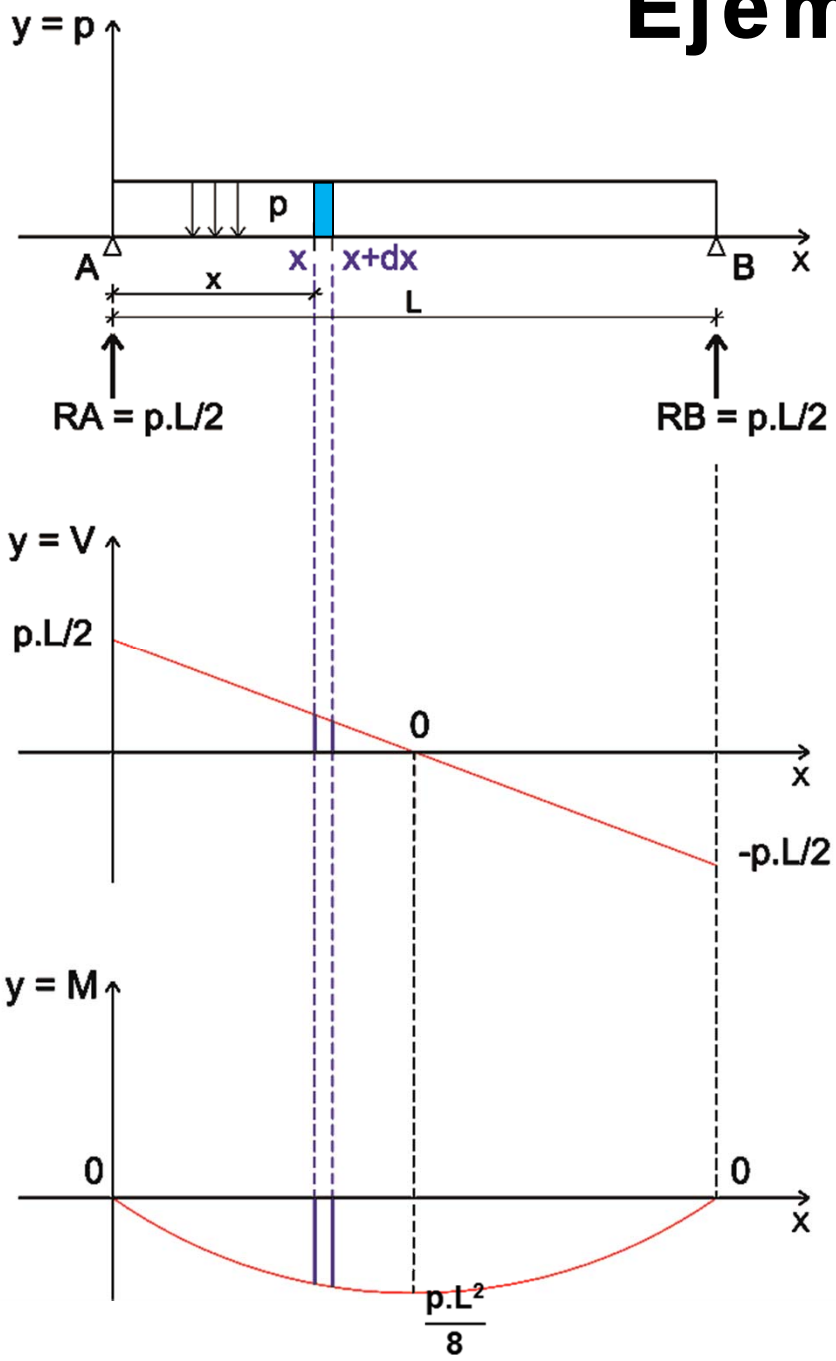
$$M`(x) = \frac{p.L}{2} - p.x$$

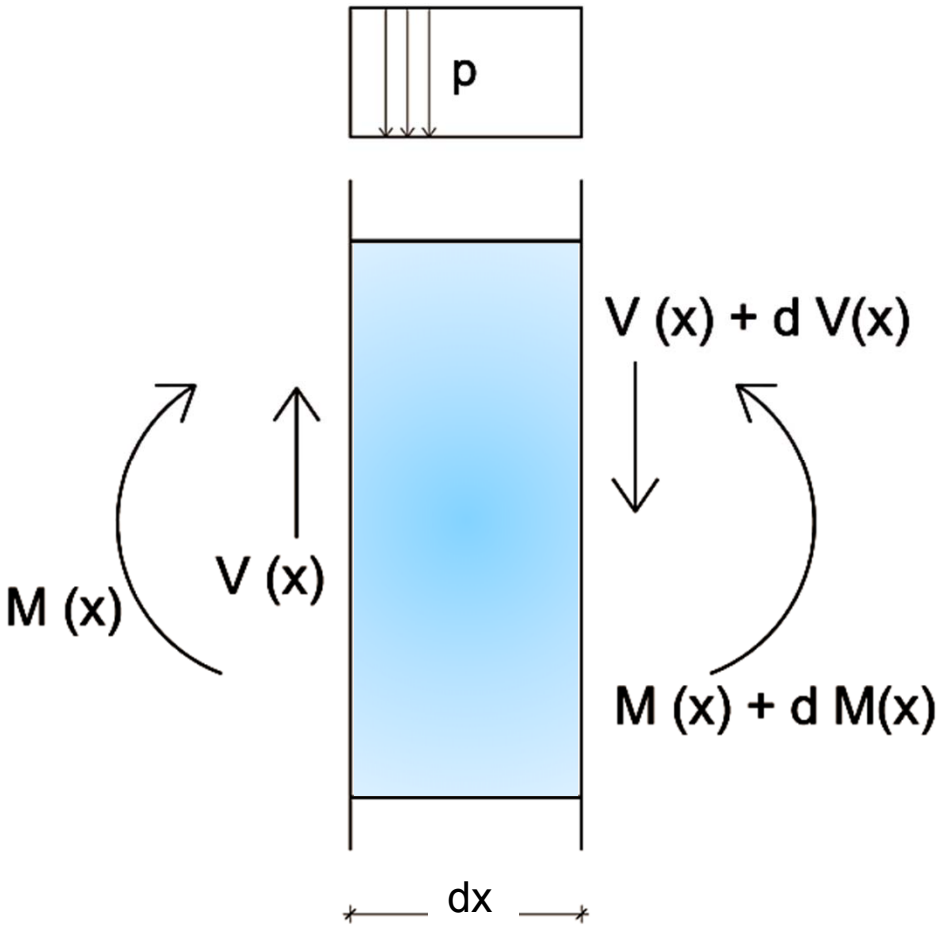
$$M`(x) = \frac{p.L}{2} - p.x = V(x)$$

$$M``(x) = -p$$

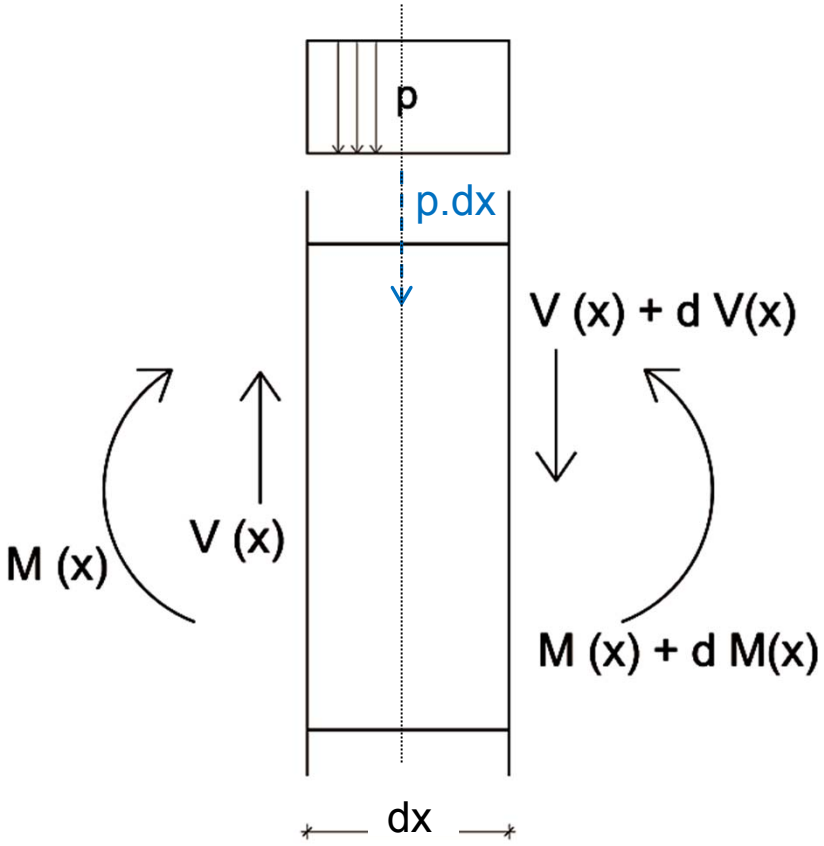
$$M``(x) = V`(x) = -p$$

Ejemplo: viga





Dovela de ancho diferencial



Equilibrio de la dovela:

$$\sum Fv = 0$$

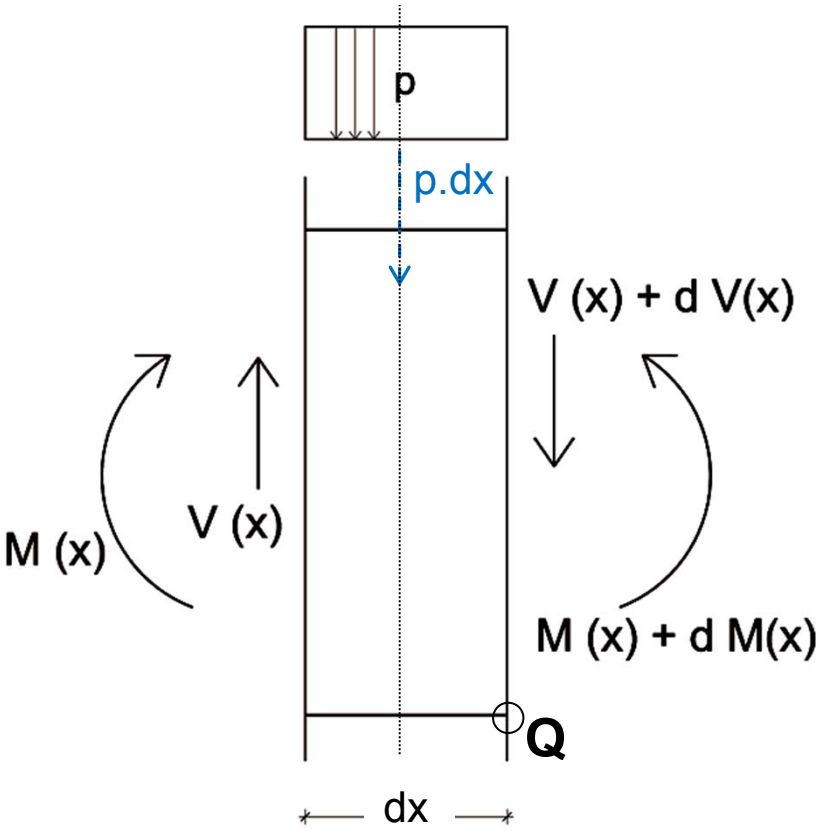
$$V(x) - p.dx - \left(V(x) + dV(x)\right) = 0$$

$$V(x) - p.dx - V(x) - dV(x) = 0$$

$$-p.dx = dV(x)$$

$$-p = \frac{dV(x)}{dx} = V`(x)$$

$$\boxed{-p = V`(x)}$$



Equilibrio de la dovela:

$$\sum M_Q = 0$$

$$M(x) - \big(M(x) + dM(x)\big) + V(x).dx - p.dx.dx/2 = 0$$

$$M(x) - M(x) - dM(x) + V(x).dx - \boxed{p.dx^2/2} = 0$$

tiende a 0

$$- dM(x) + V(x).dx = 0$$

$$V(x).dx = dM(x)$$

$$V(x) = \frac{dM(x)}{dx} = M`(x) \qquad \boxed{V(x) = M`(x)}$$

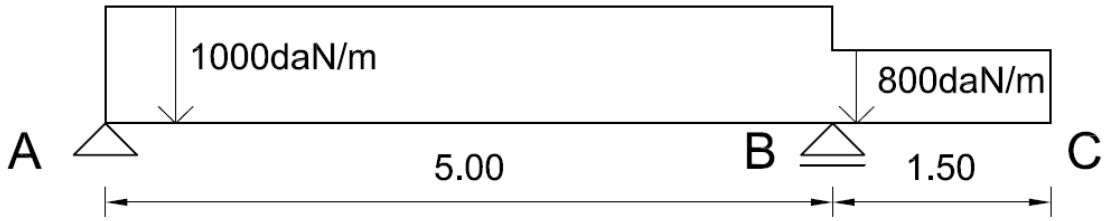
$$-p = V'(x)$$

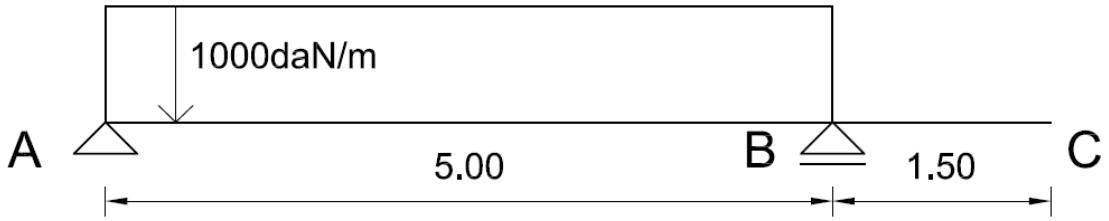
$$V(x) = M'(x)$$

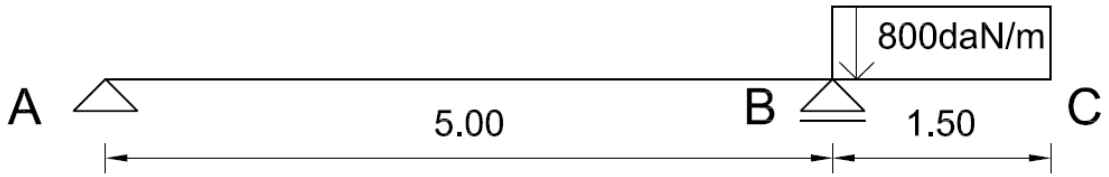
$$\boxed{M''(x) = V'(x) = -p}$$

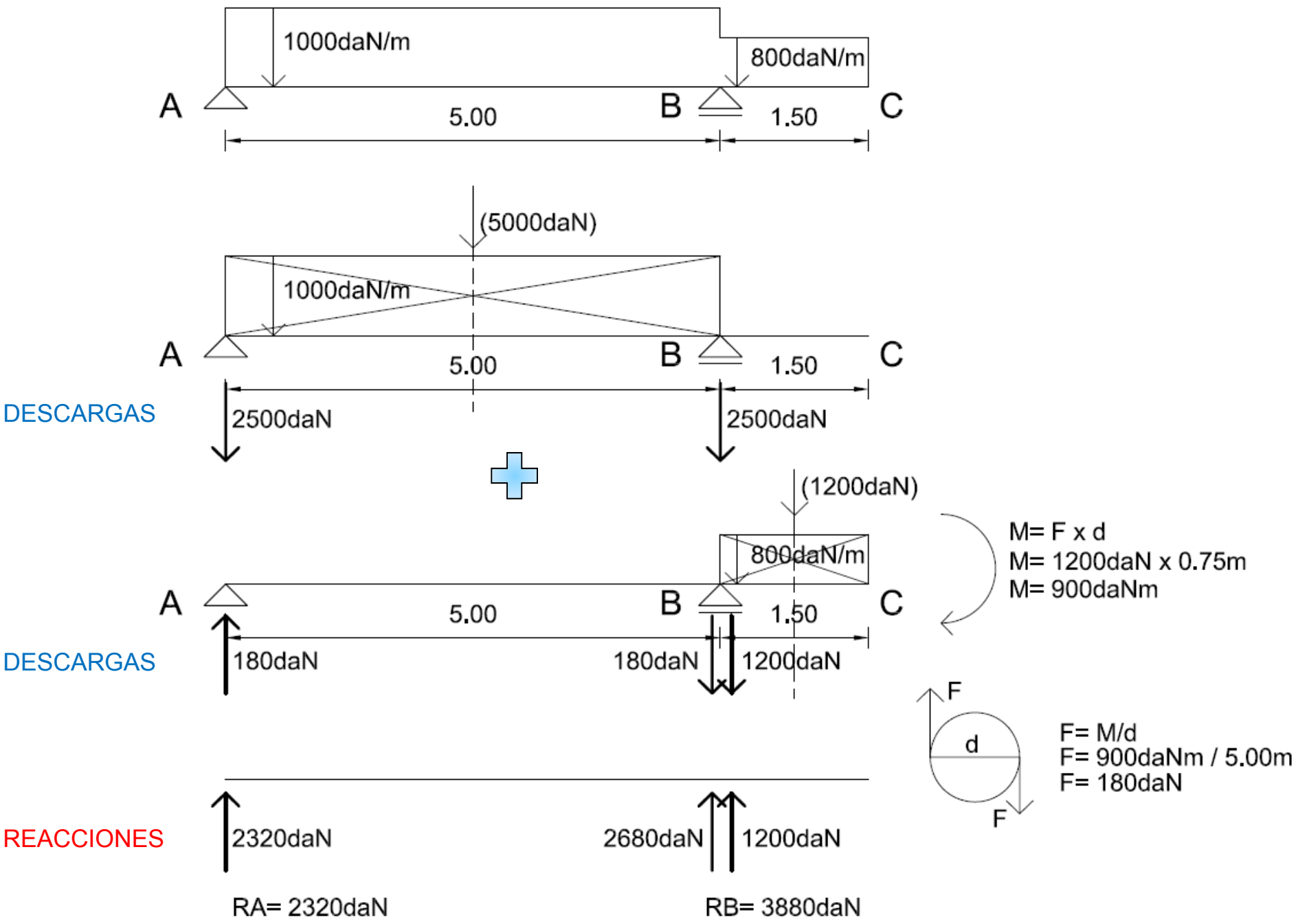
ecuación fundamental de las vigas rectas

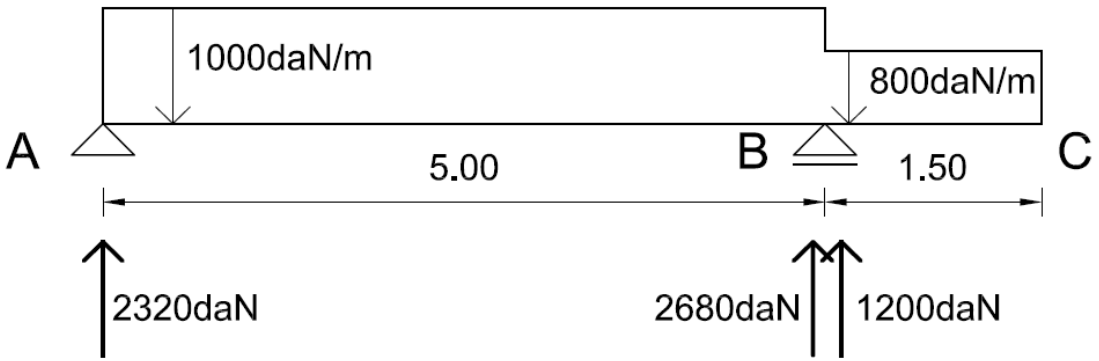
EJEMPLO PRÁCTICO


1000daN/m
800daN/m
A
B
C
5.00
1.50


1000daN/m
A
B
C
5.00
1.50


800daN/m
A
B
C
5.00
1.50


1000daN/m
800daN/m
A
B
C
5.00
1.50
(5000daN)
1000daN/m
A
B
C
5.00
1.50
DESCARGAS
2500daN
2500daN
(1200daN)
800daN/m
A
B
C
5.00
1.50
DESCARGAS
180daN
180daN
1200daN
M= F x d
M= 1200daN x 0.75m
M= 900daNm
F
d
F
F= M/d
F= 900daNm / 5.00m
F= 180daN
REACCIONES
2320daN
2680daN
1200daN
RA= 2320daN
RB= 3880daN



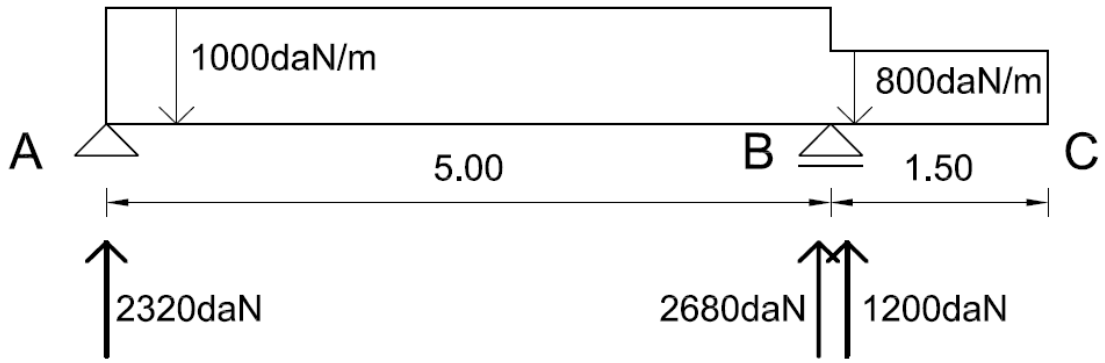
RA= 2320daN

RB= 3880daN

$$\Sigma F_V = 0$$

1000daN/m . 5,00m + 800daN/m . 1,50m - 2320daN - 3880daN = 0

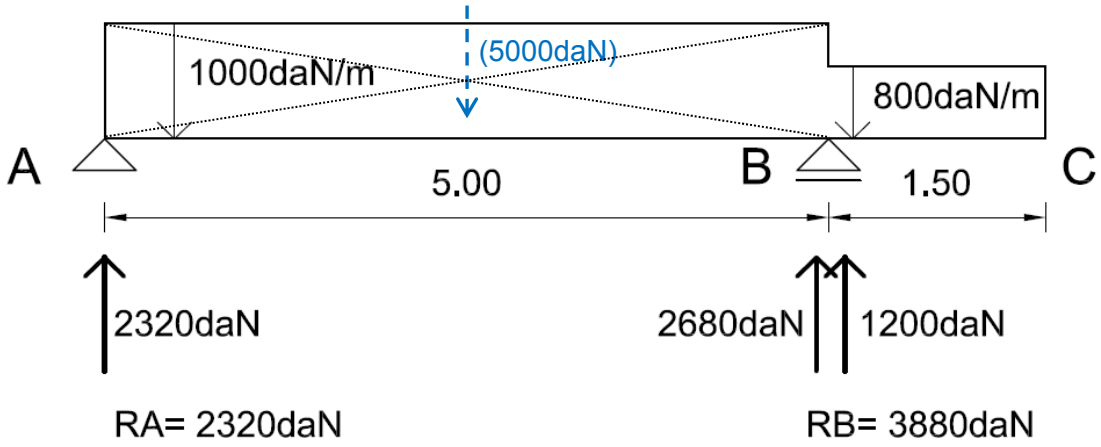
5000daN + 1200daN - 2320daN - 3880daN = 0



RA= 2320daN

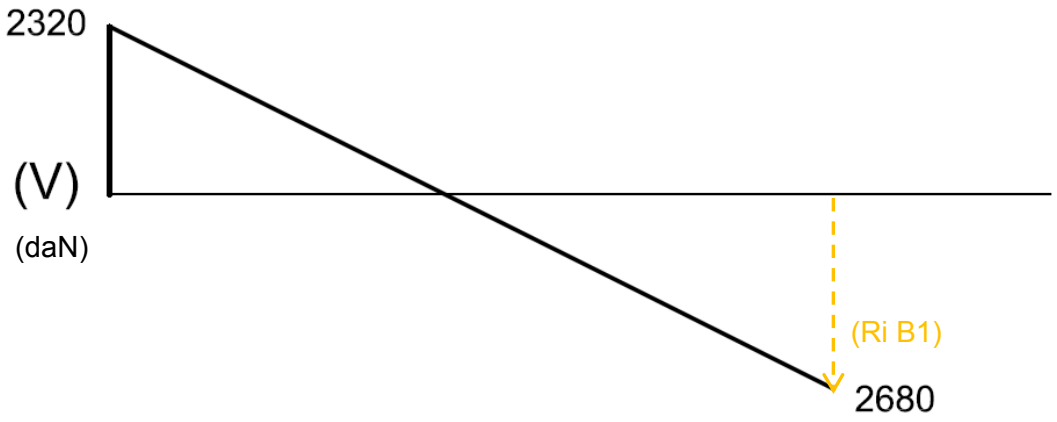
RB= 3880daN

Rizq A = RA = 2320daN



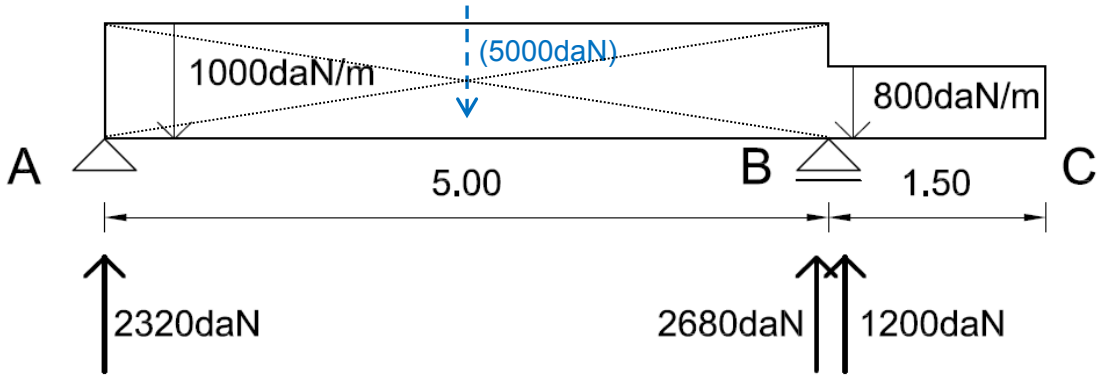
RA= 2320daN

RB= 3880daN



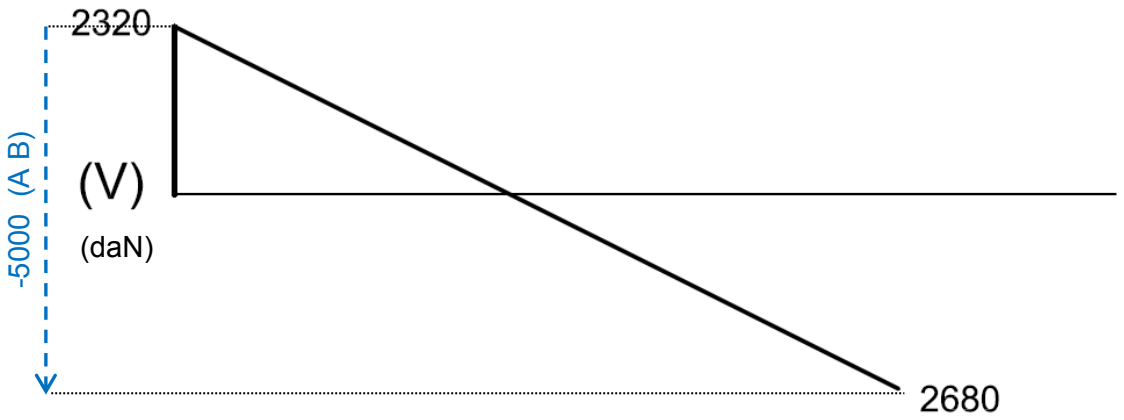
Rizq A = RA = 2320daN

Rizq B1 = RA - 5000daN = -2680daN



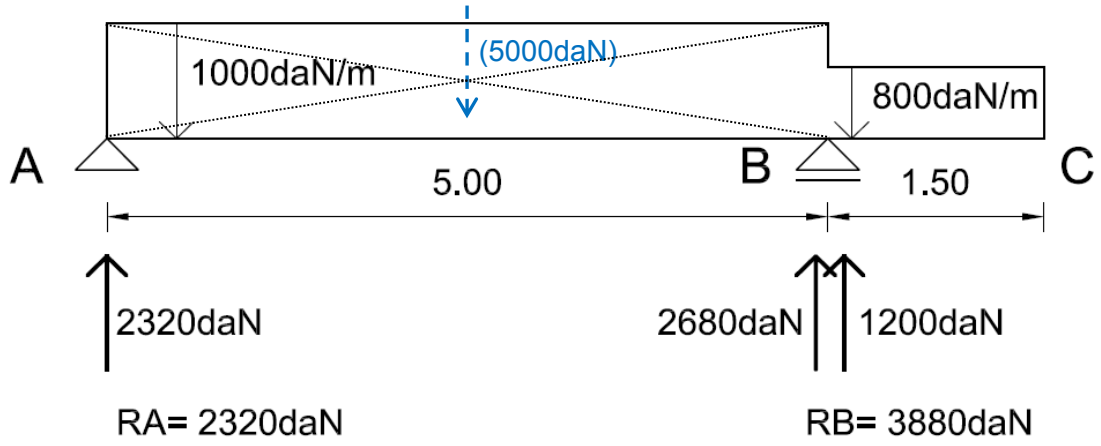
RA= 2320daN

RB= 3880daN



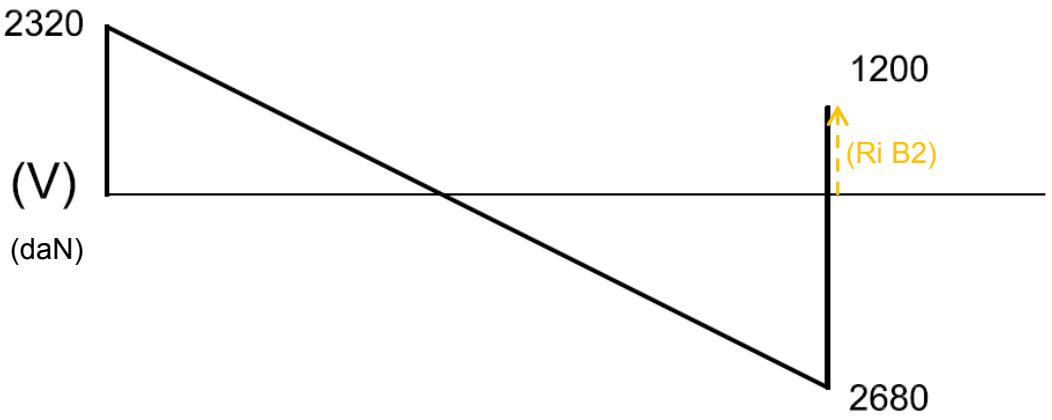
Rizq A = RA = 2320daN

Rizq B1 = RA - 5000daN = -2680daN



RA= 2320daN

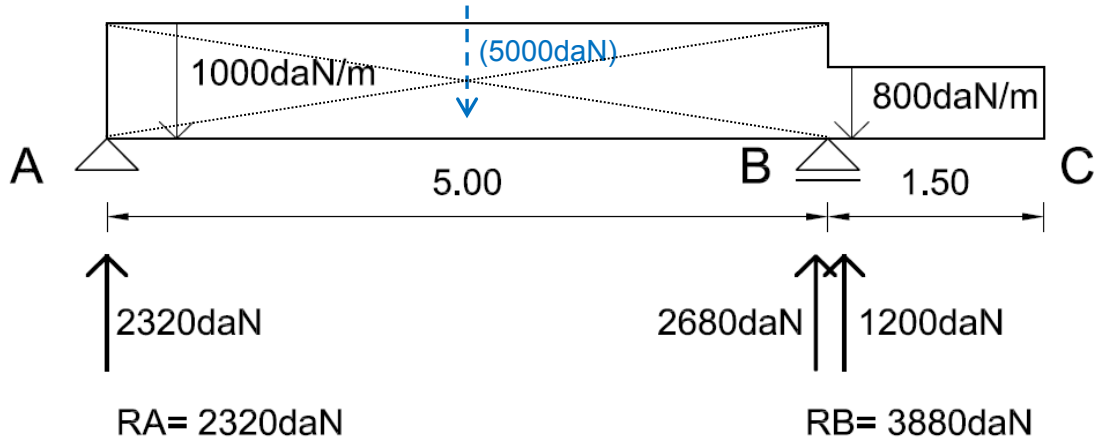
RB= 3880daN



Rizq A = RA = 2320daN

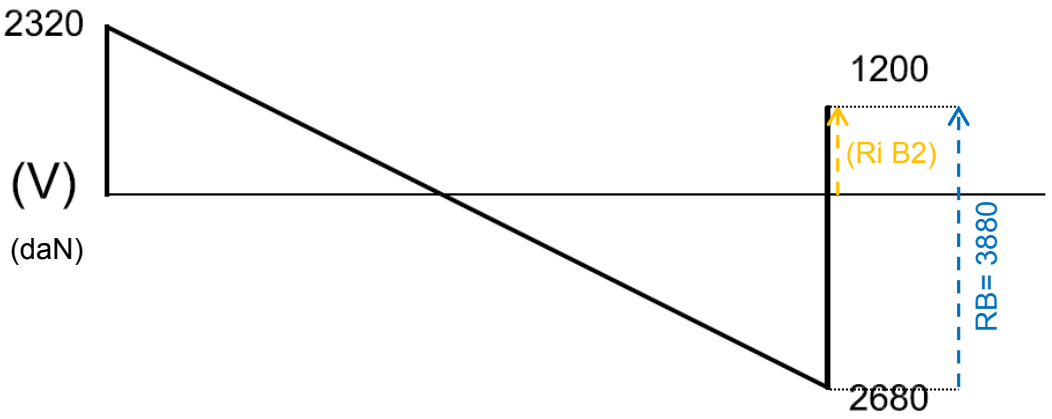
Rizq B1 = RA - 5000daN = -2680daN

Rizq B2 = RA - 5000daN + RB = 1200daN



RA= 2320daN

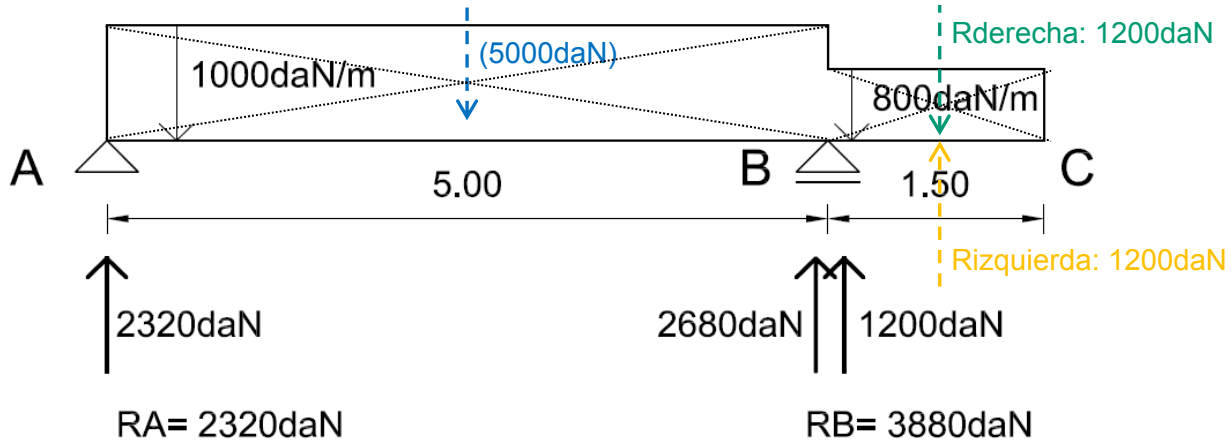
RB= 3880daN



Rizq A = RA = 2320daN

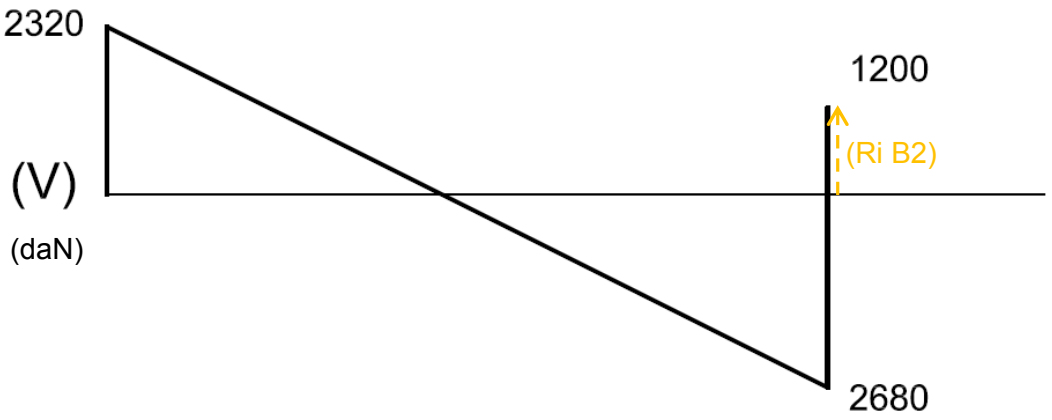
Rizq B1 = RA - 5000daN = -2680daN

Rizq B2 = RA - 5000daN + RB = 1200daN



RA= 2320daN

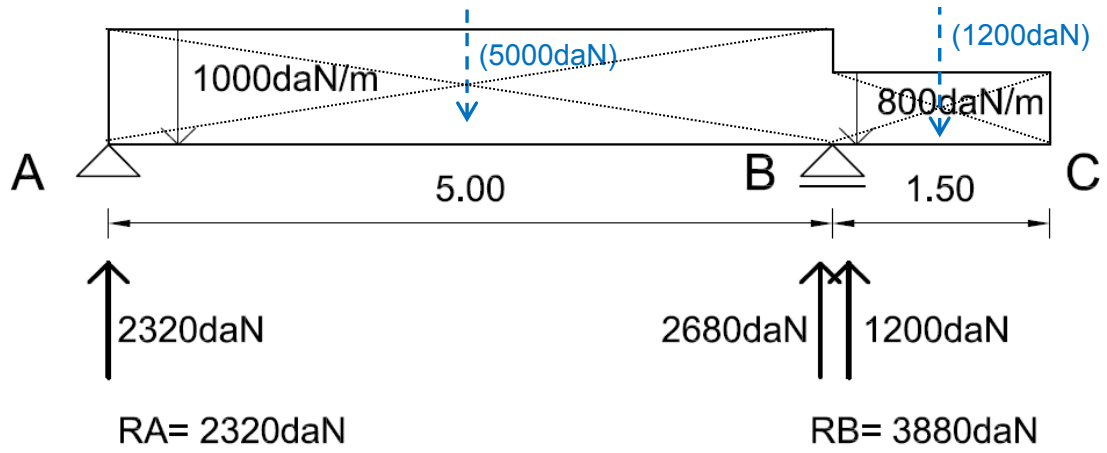
RB= 3880daN



Rizq A = RA = 2320daN

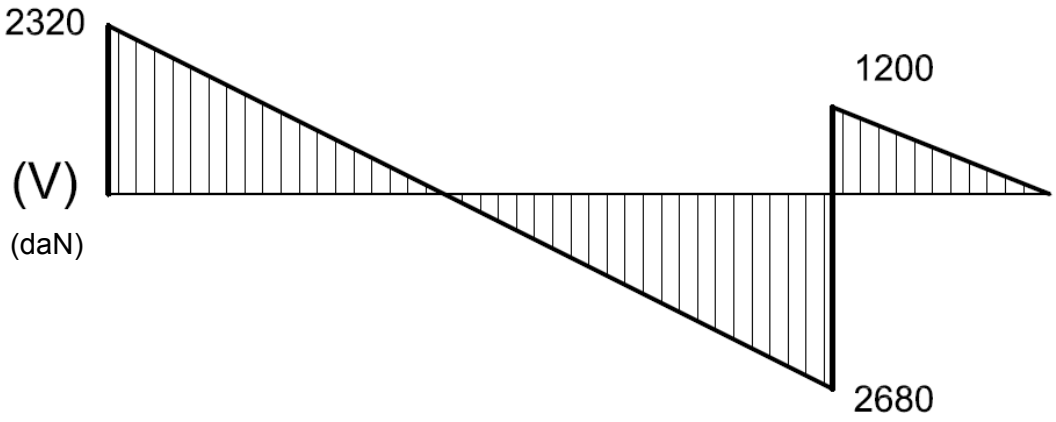
Rizq B1 = RA - 5000daN = -2680daN

Rizq B2 = RA - 5000daN + RB = 1200daN



RA= 2320daN

RB= 3880daN

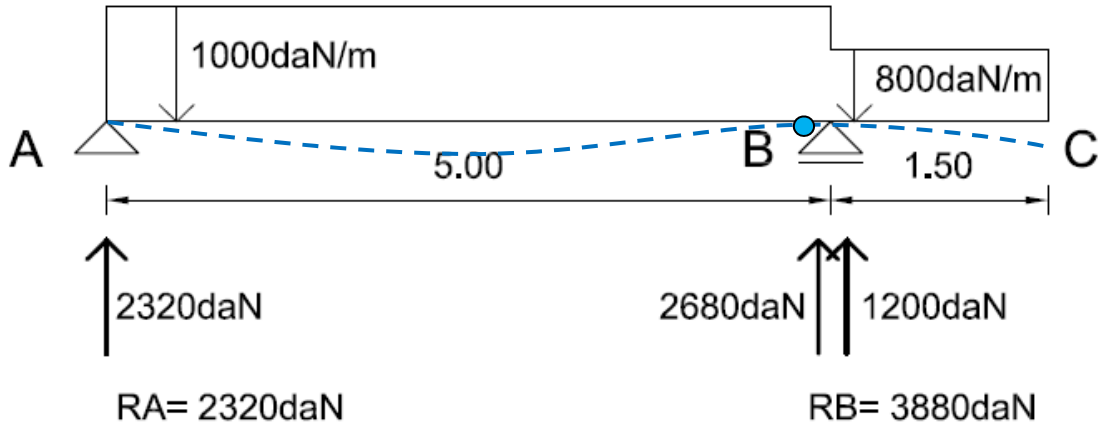


Rizq A = RA = 2320daN

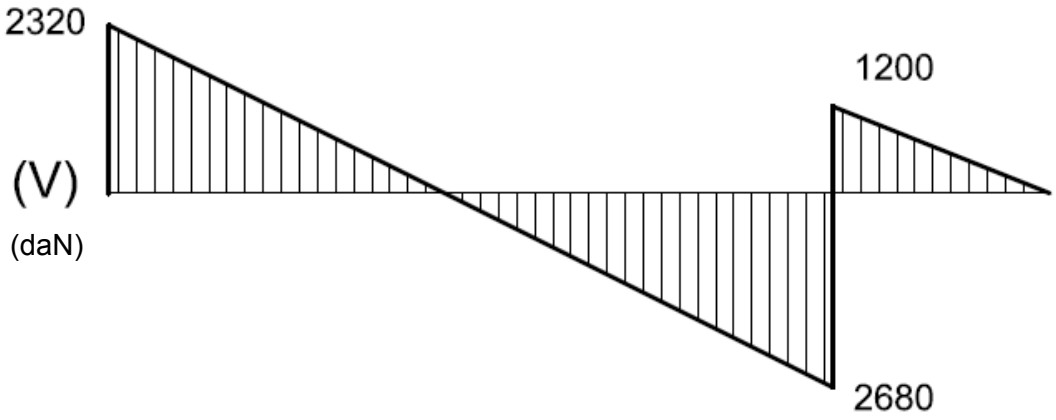
Rizq B1 = RA - 5000daN = -2680daN

Rizq B2 = RA - 5000daN + RB = 1200daN

Rizq C = RA - 5000daN + RB - 1200daN = 0

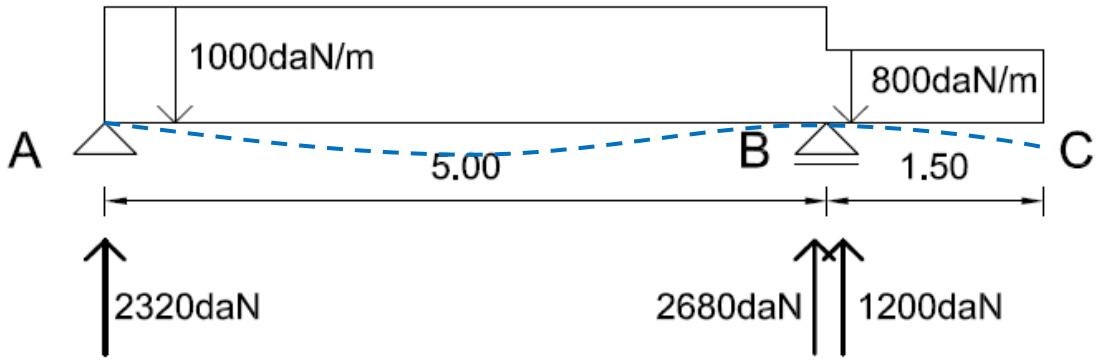


DEFORMACIÓN



(M)

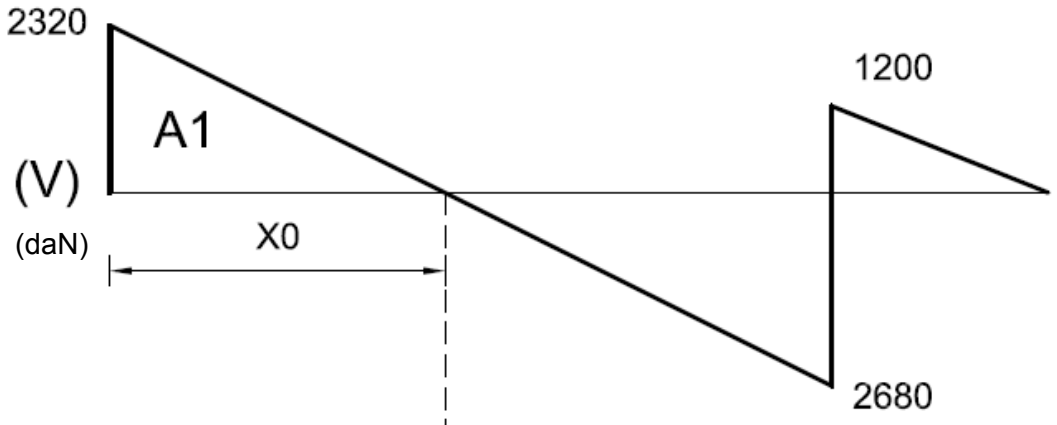
(daNm)



DEFORMACIÓN

RA= 2320daN

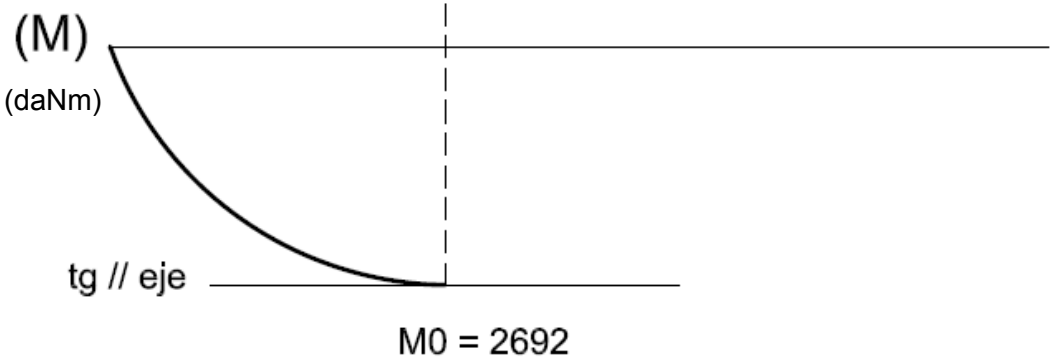
RB= 3880daN



$$X0 = \frac{V}{p}$$

$$X0 = \frac{2320daN}{1000daN/m}$$

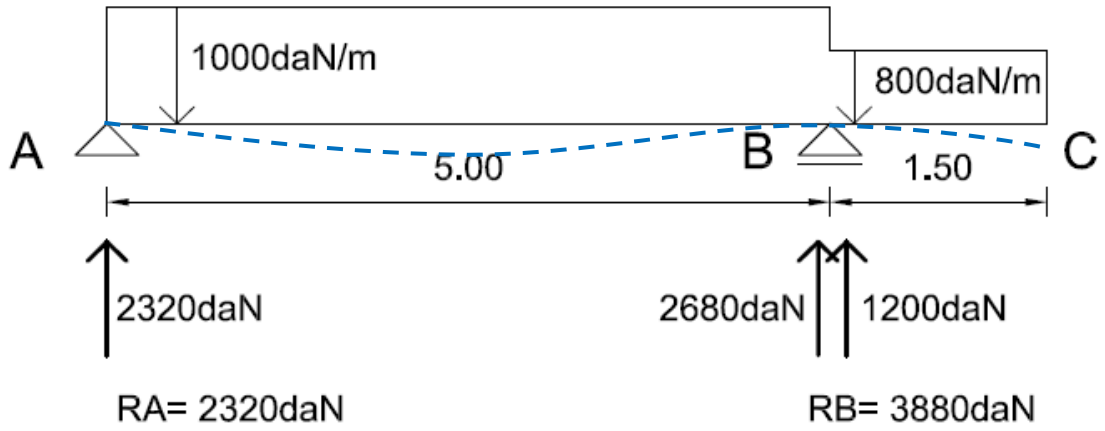
$$X0 = 2.32m$$



$$MA = 0$$

$$M0 = A1 = \frac{2320daN \times 2.32m}{2}$$

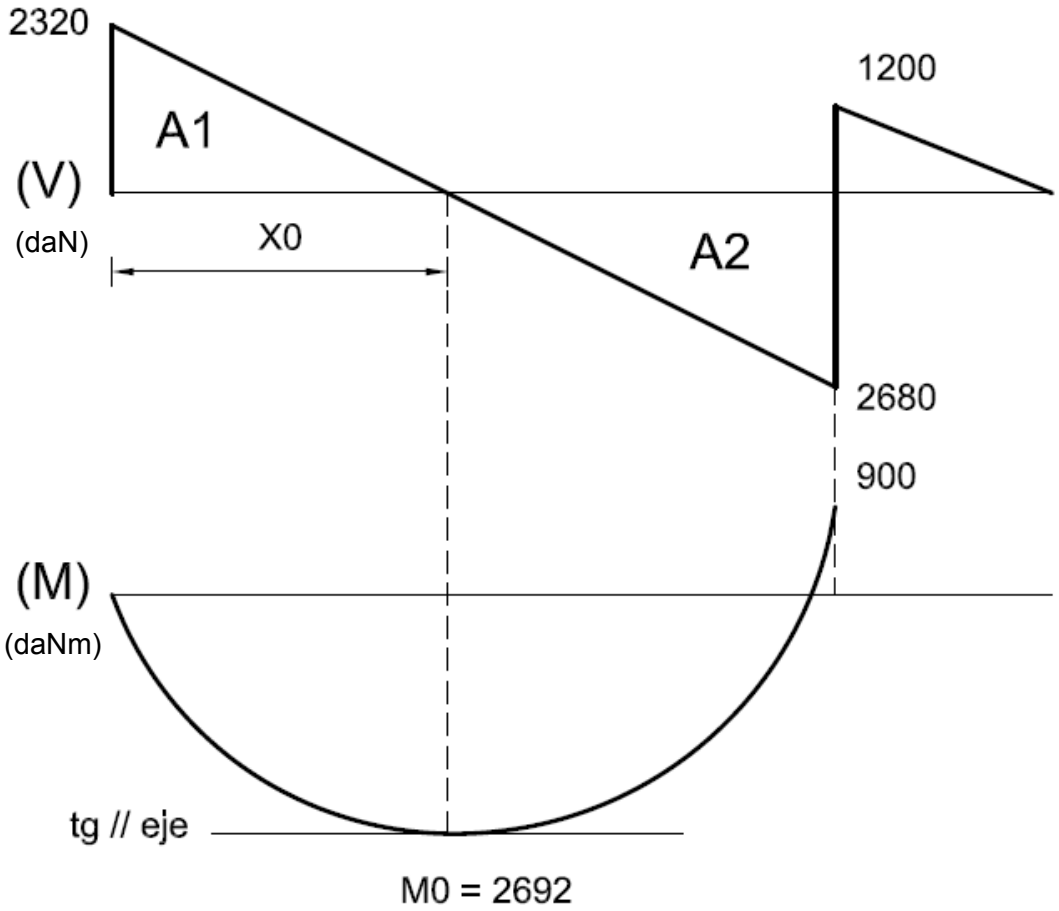
$$M0 = 2692daNm$$



DEFORMACIÓN

RA= 2320daN

RB= 3880daN



$$X0 = \frac{V}{p}$$

$$X0 = \frac{2320daN}{1000daN/m}$$

X0 = 2.32m

MA = 0

M0 = A1 = 2692daNm

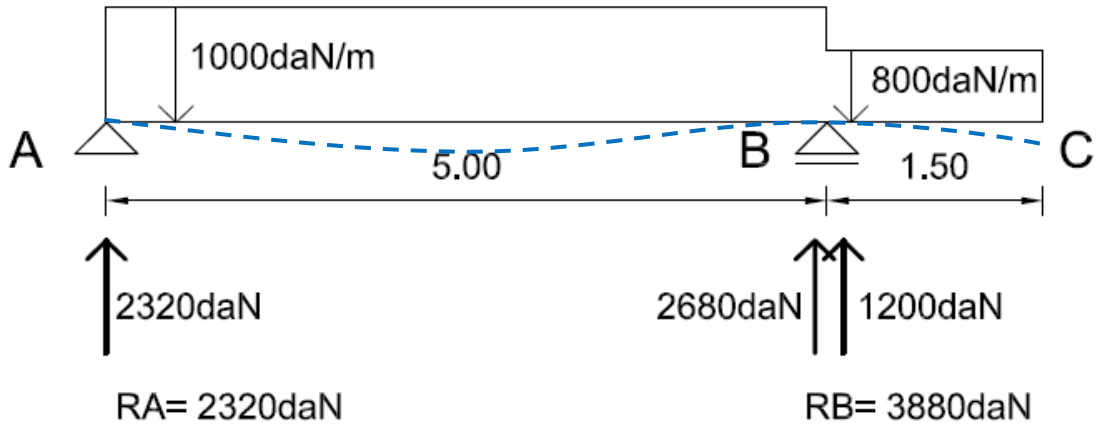
MB = A1 - A2 = -900daNm

MB = A1 - A2 = A3

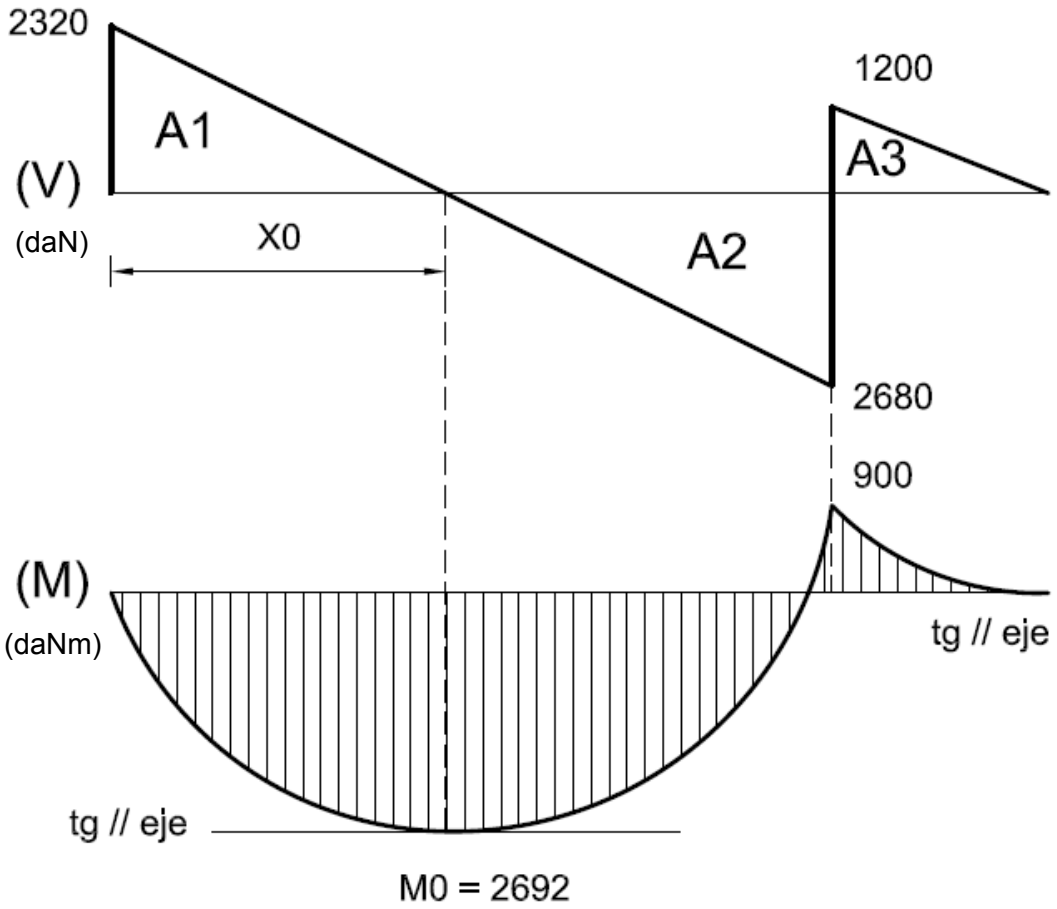
M= F x d

M= 1200daN x 0.75m

M= 900daNm



DEFORMACIÓN



$$X0 = \frac{V}{p}$$

$$X0 = \frac{2320daN}{1000daN/m}$$

$X0 = 2.32m$

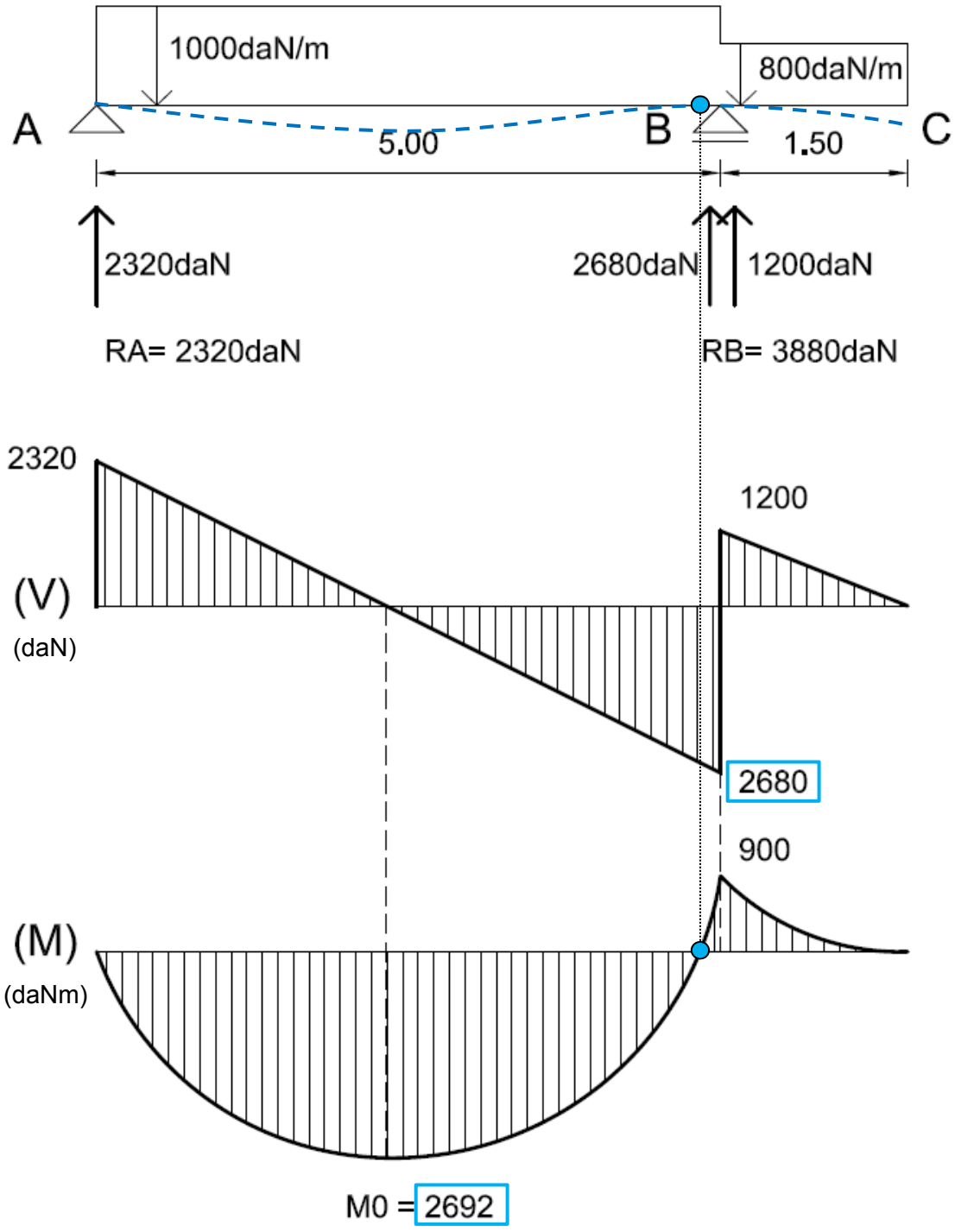
$MA = 0$

$M0 = A1 = 2692daNm$

$MB = A1 - A2 = -900daNm$

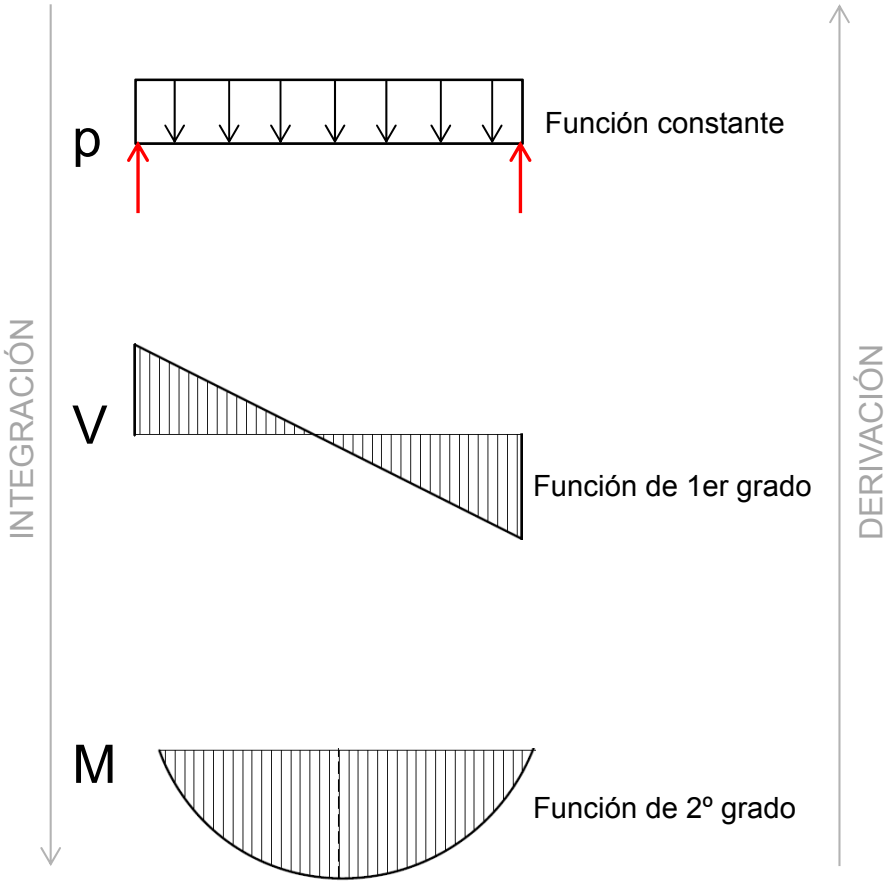
$MB = A1 - A2 = A3$

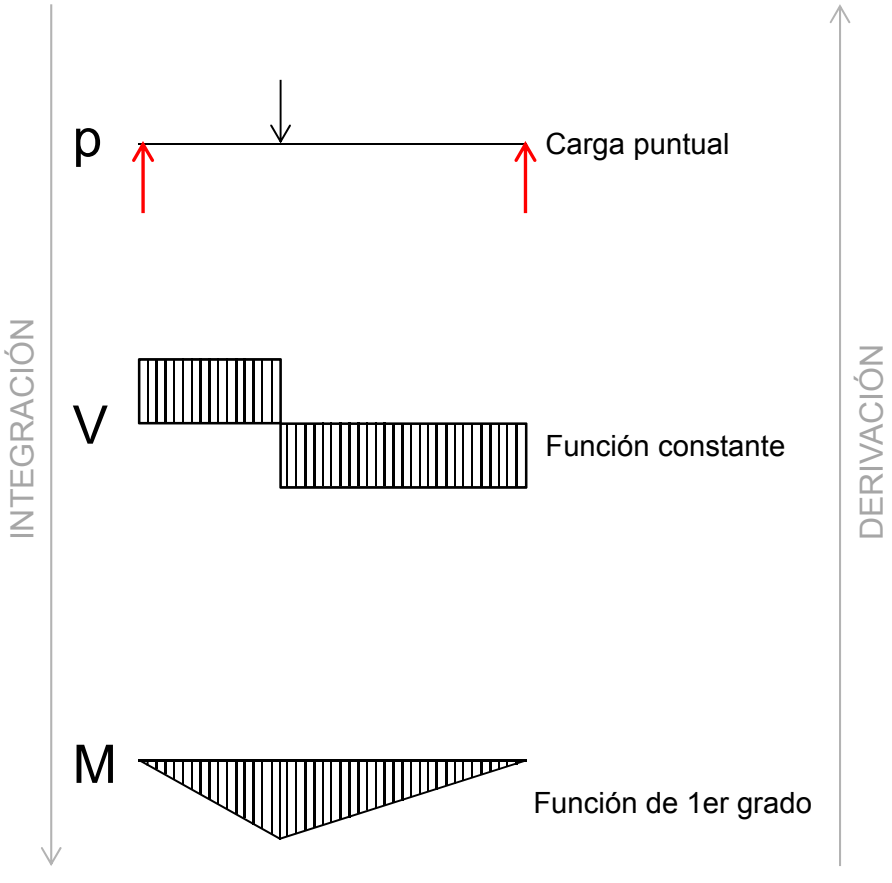
$MC = A1 - A2 + A3 = 0$



DEFORMACIÓN

p / V / M


INTEGRACIÓN
DERIVACIÓN
p
Función constante
V
Función de 1er grado
M
Función de 2º grado


INTEGRACIÓN
DERIVACIÓN
p
Carga puntual
V
Función constante
M
Función de 1er grado