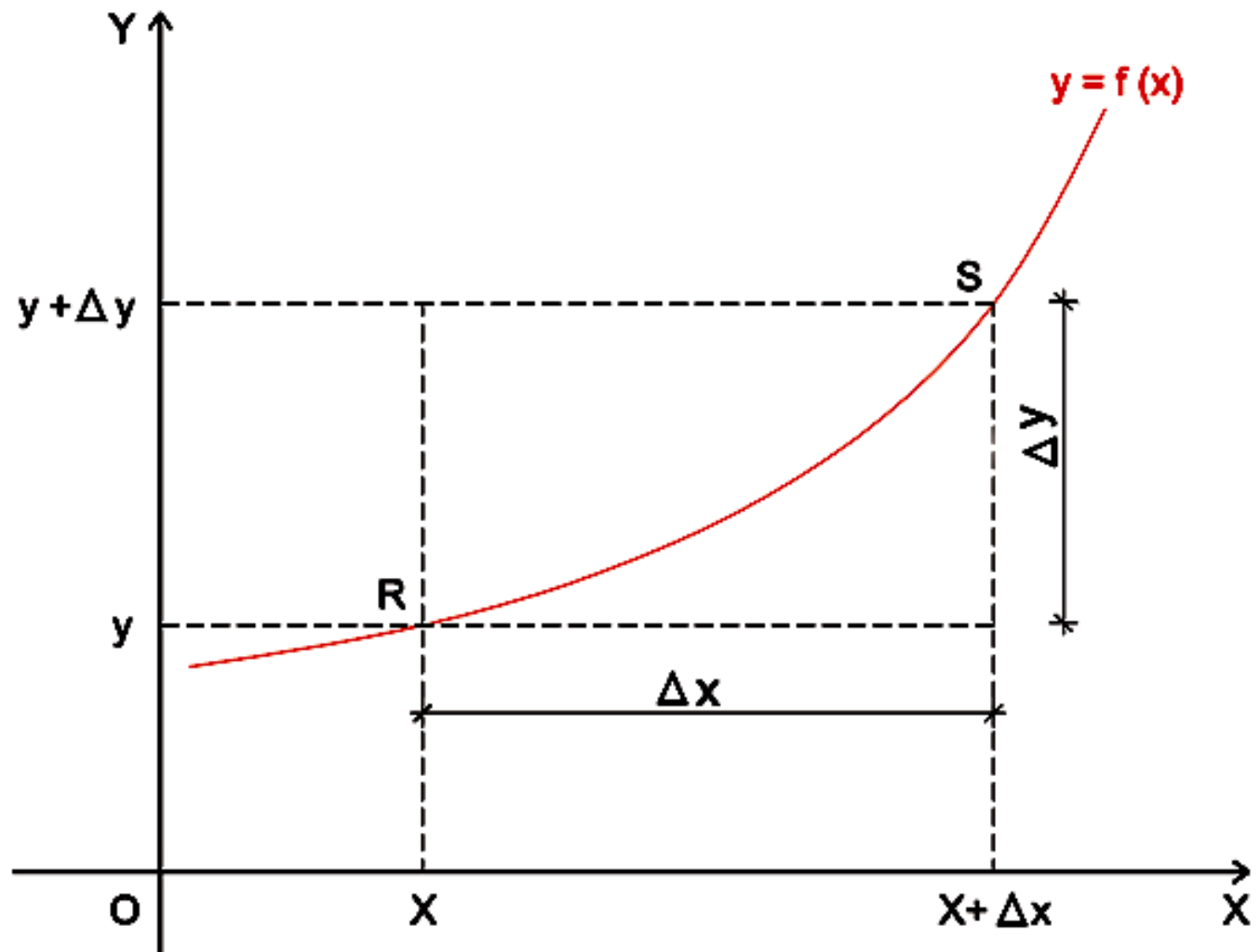


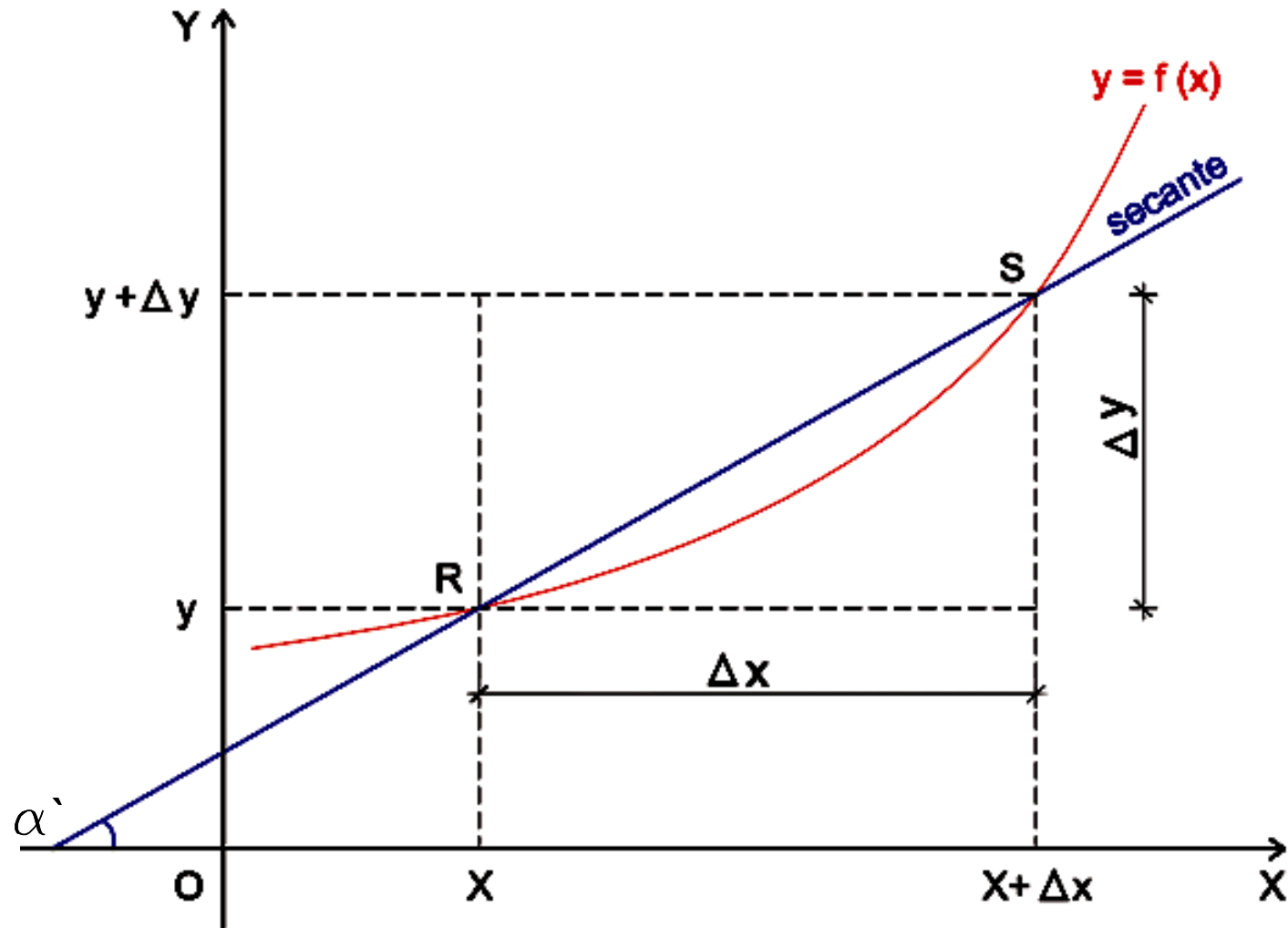
TERCERA FAMILIA de ESTRUCTURAS: ESTRUCTURAS FLEXADAS

2. RELACIONES entre CARGA (p), CORTANTE (V) y MOMENTO (M).

DERIVADA



DERIVADA



$\text{tg } \alpha' = \Delta y / \Delta x$ (coeficiente angular o pendiente de la recta secante)

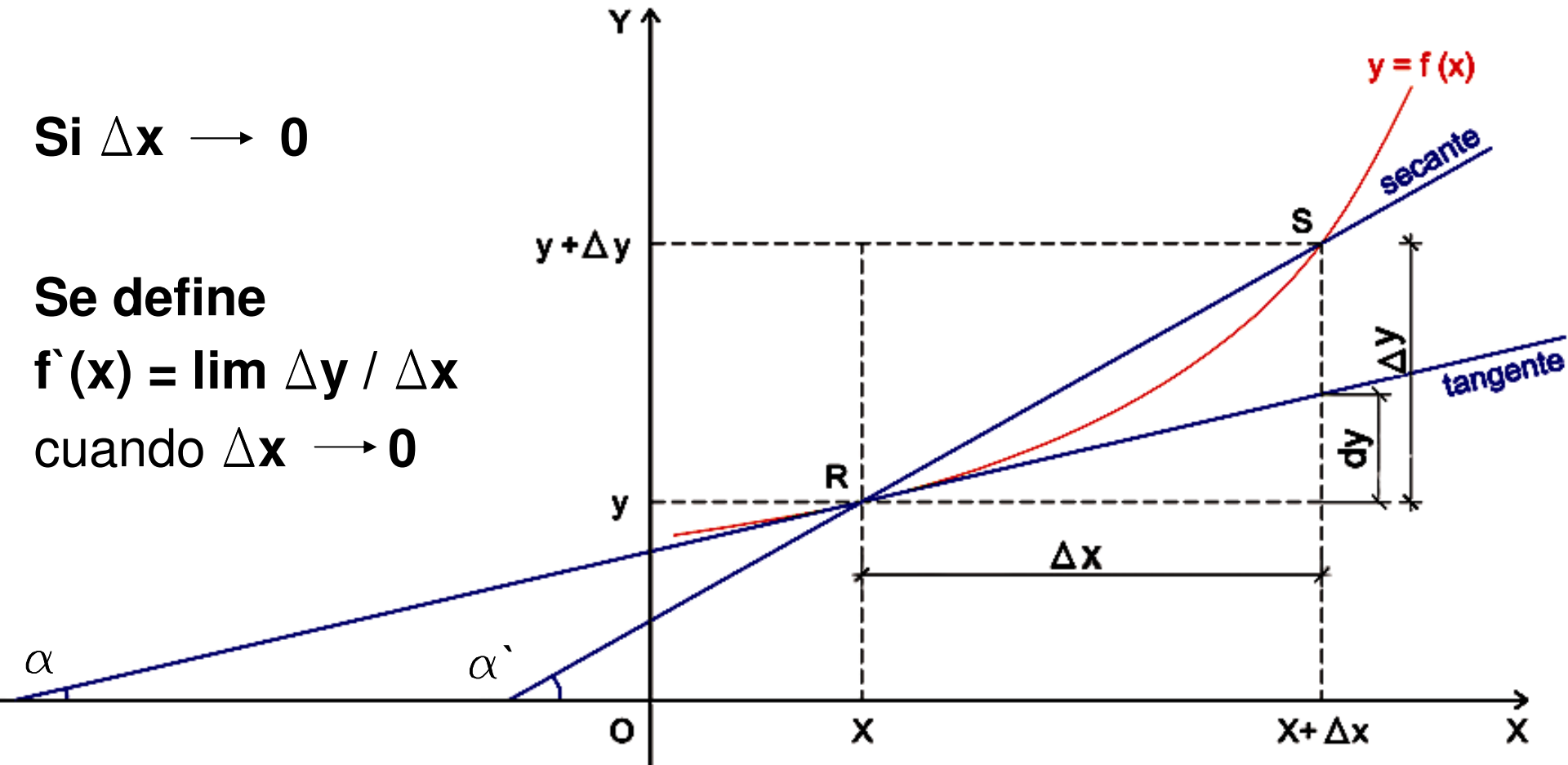
DERIVADA

Si $\Delta x \rightarrow 0$

Se define

$$f'(x) = \lim \Delta y / \Delta x$$

cuando $\Delta x \rightarrow 0$



La derivada en un punto representa el coeficiente angular o pendiente de la recta tangente a la función en ese punto.

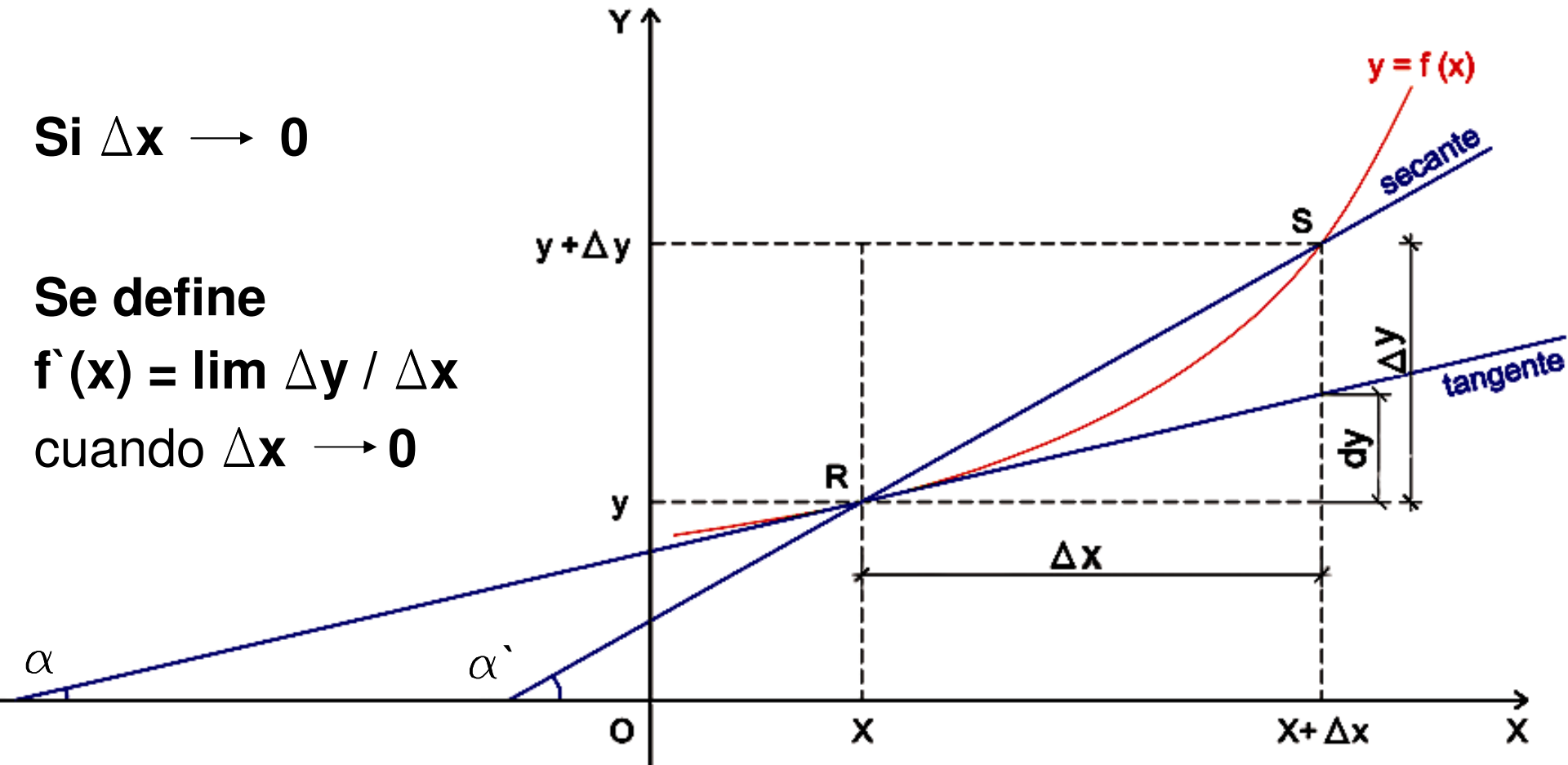
DERIVADA

Si $\Delta x \rightarrow 0$

Se define

$$f'(x) = \lim \Delta y / \Delta x$$

cuando $\Delta x \rightarrow 0$



$$\text{tg } \alpha = f'(x) = \boxed{dy / dx = y'(x)}$$

$$dx = \Delta x \quad \text{si } \Delta x \rightarrow 0$$

INTEGRAL INDEFINIDO

F (x) es primitiva de f (x) si $F'(x) = f(x)$
es la operación contraria a la derivación

Ejemplo: $f(x) = x^2$

$f'(x) = 2 \cdot x$

$F(x) = x^3/3 + c$

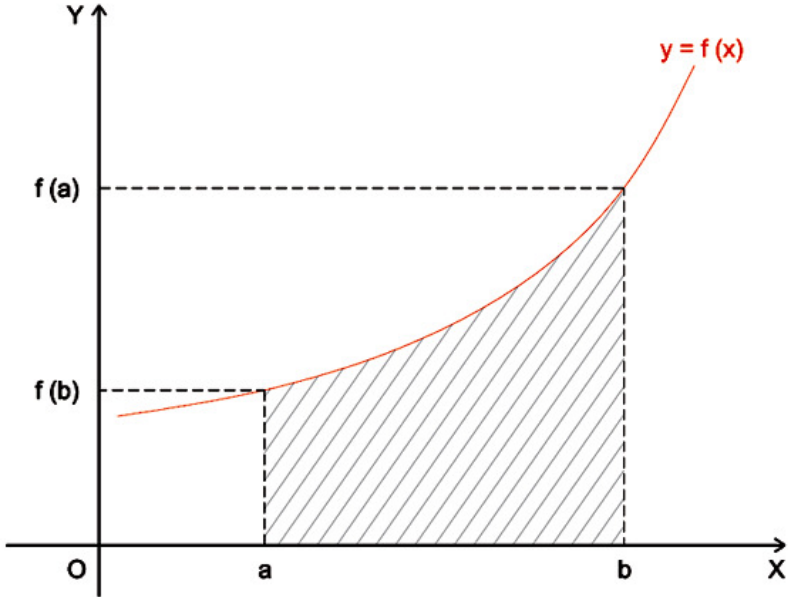
$f(x) = x^n$

$f'(x) = n \cdot x^{n-1}$

$F(x) = (x^{n+1} / n+1) + c$

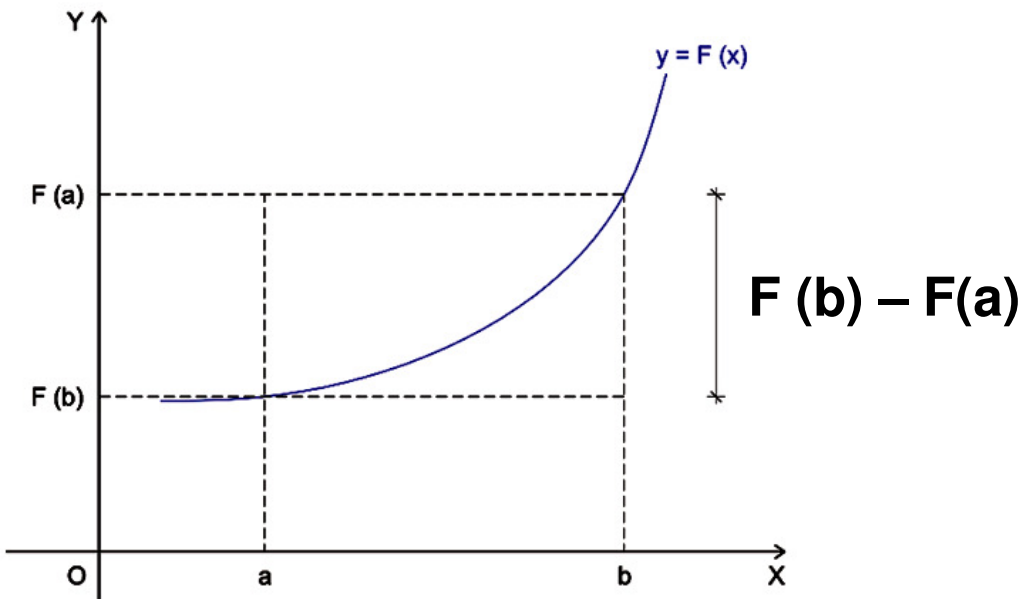
$$\int f(x) dx = F(x) + c$$

INTEGRAL DEFINIDO

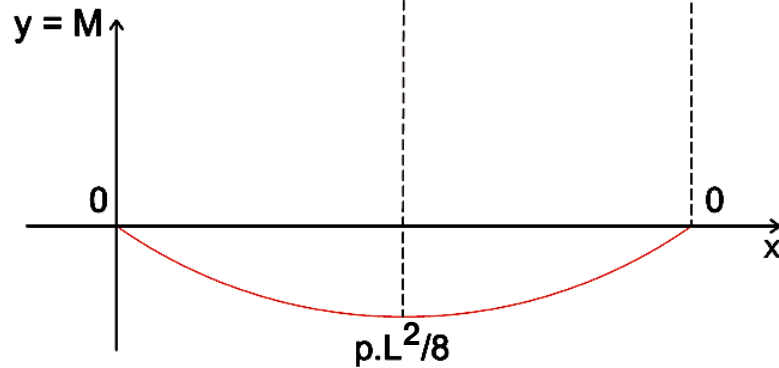
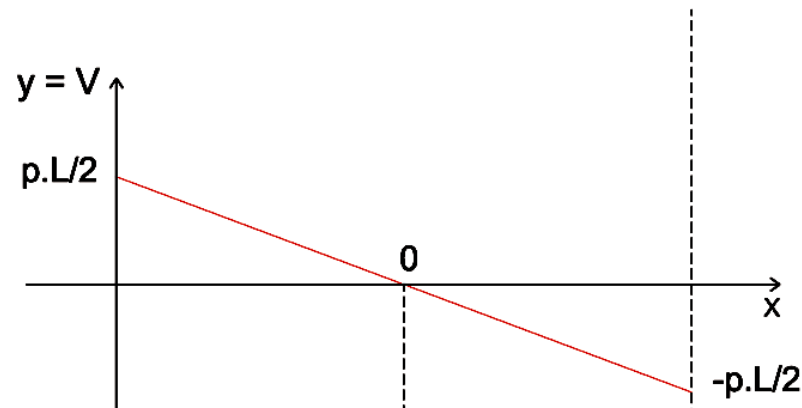
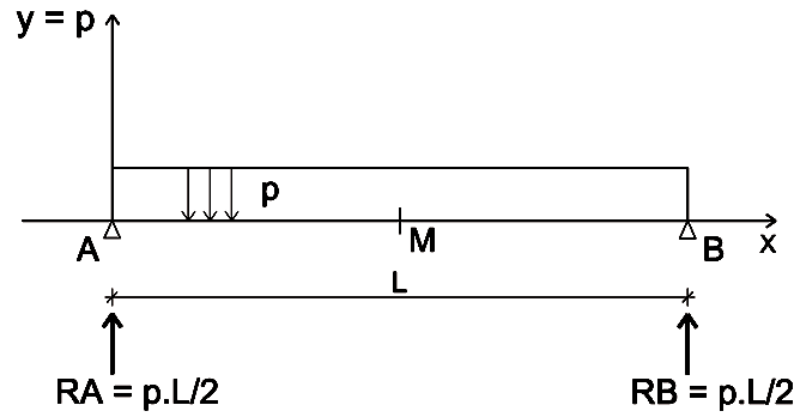


$$\int_a^b f(x) dx = F(b) - F(a)$$

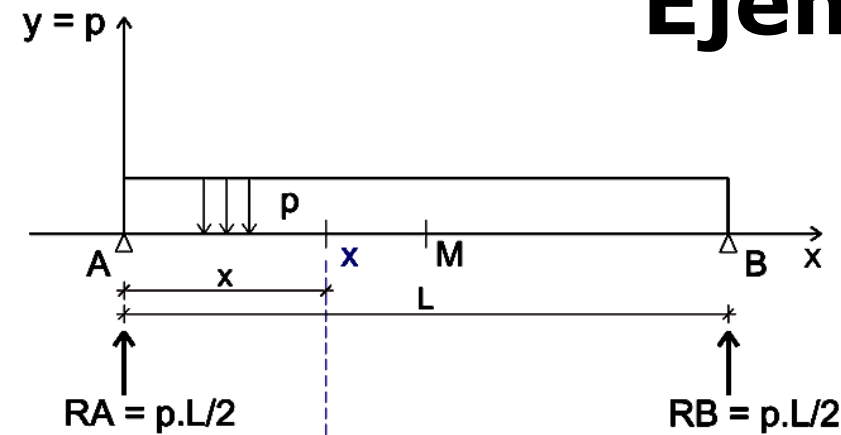
siendo $F(x)$ una primitiva de $f(x)$



Ejemplo: viga



Ejemplo: viga

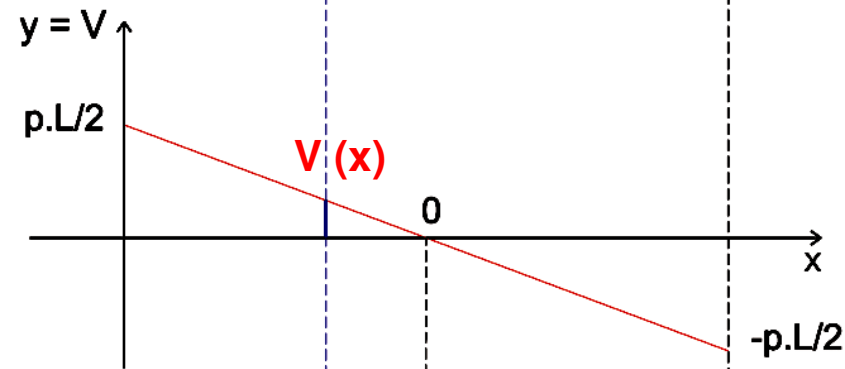


$$V(x) = p.L / 2 - p.x$$

$$V(0) = p.L / 2$$

$$V(L/2) = 0$$

$$V(L) = -p.L / 2$$

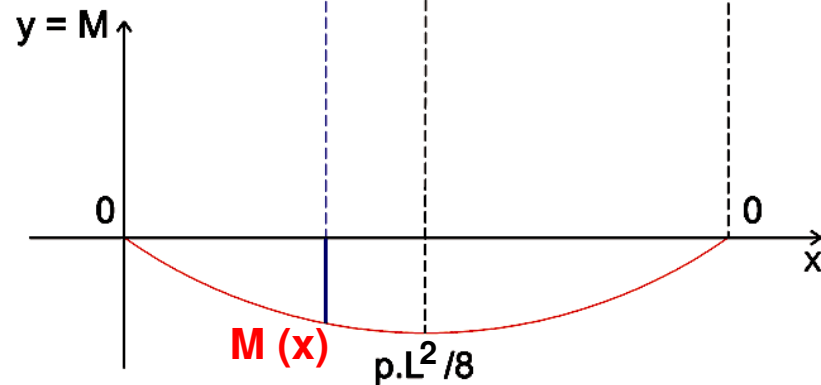


$$M(x) = (p.L/2).x - p.x^2 / 2$$

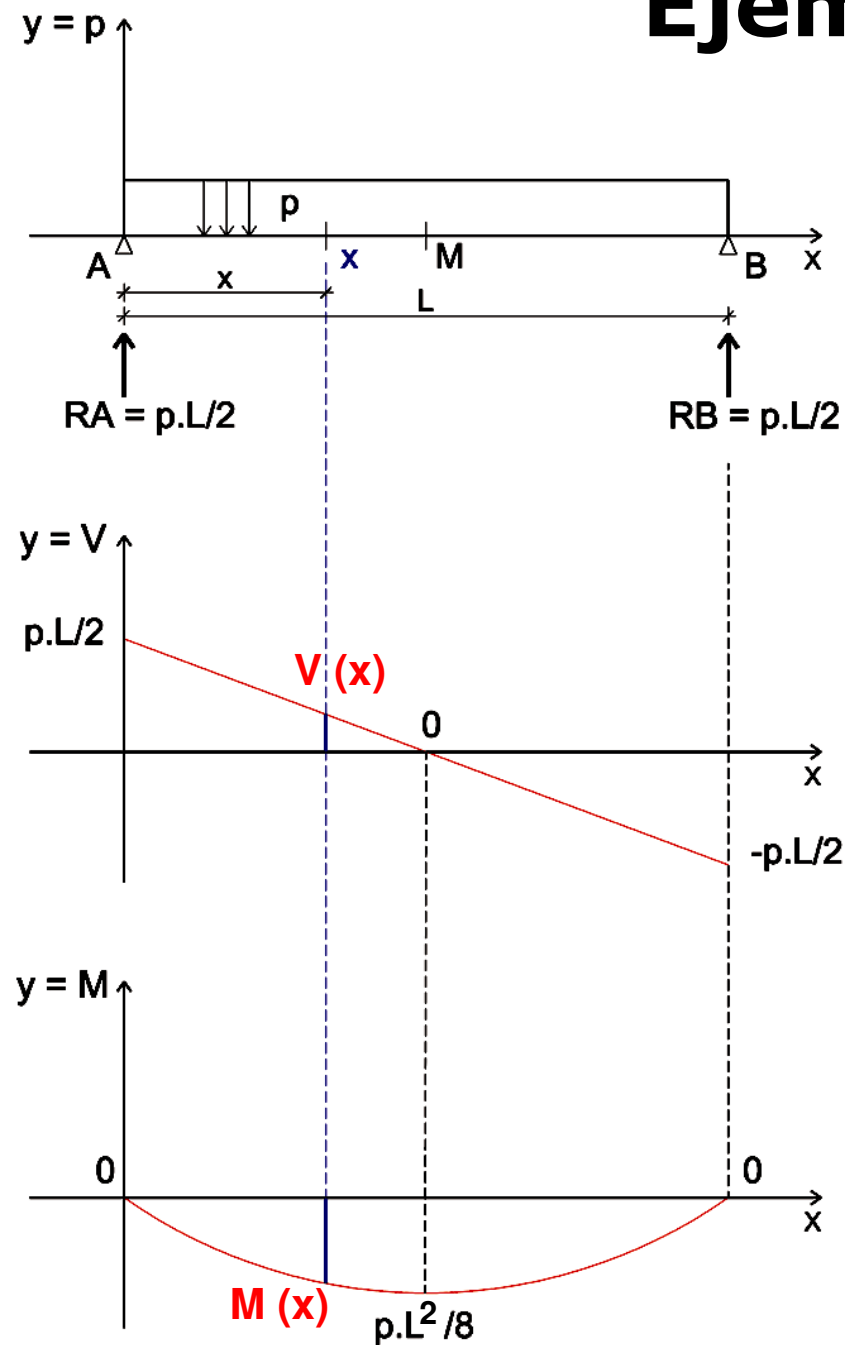
$$M(0) = 0$$

$$M(L/2) = p.L^2 / 8$$

$$M(L) = 0$$



Ejemplo: viga



$$M(x) = (p.L/2).x - p.x^2 / 2$$

$$M'(x) = p.L / 2 - (2.p/2).x$$

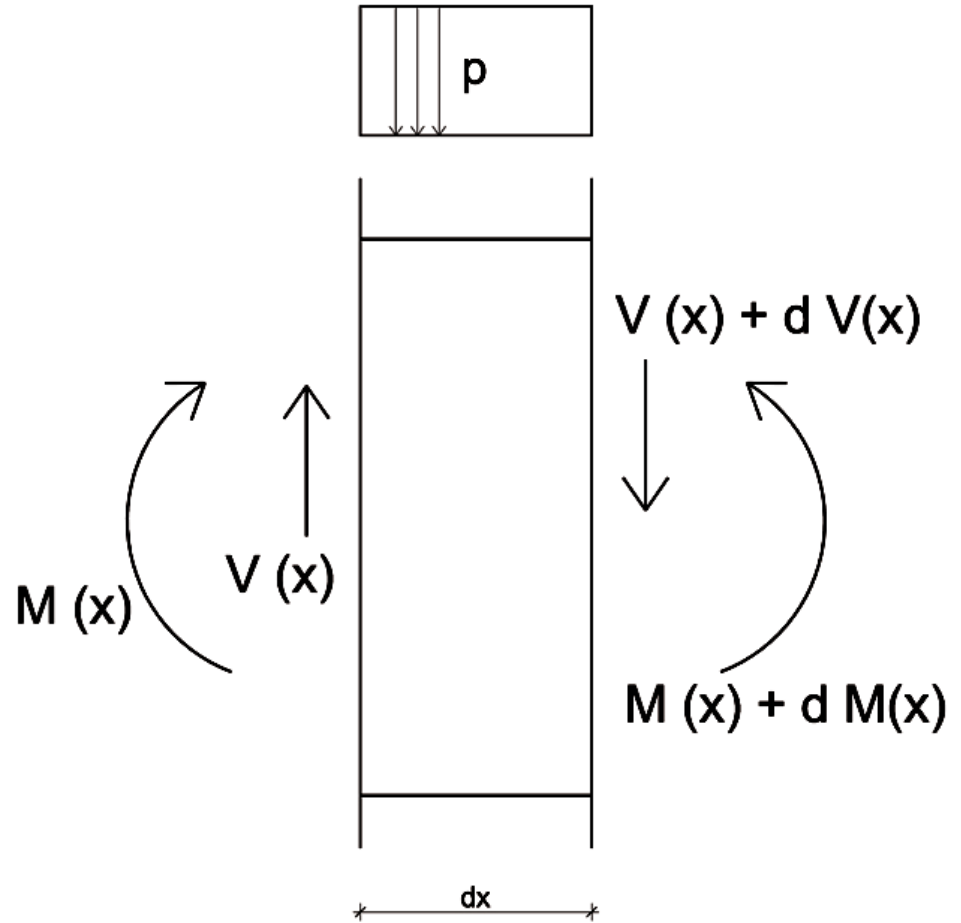
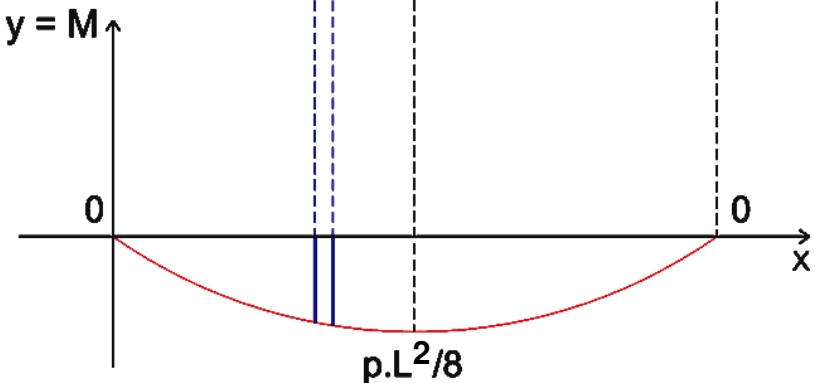
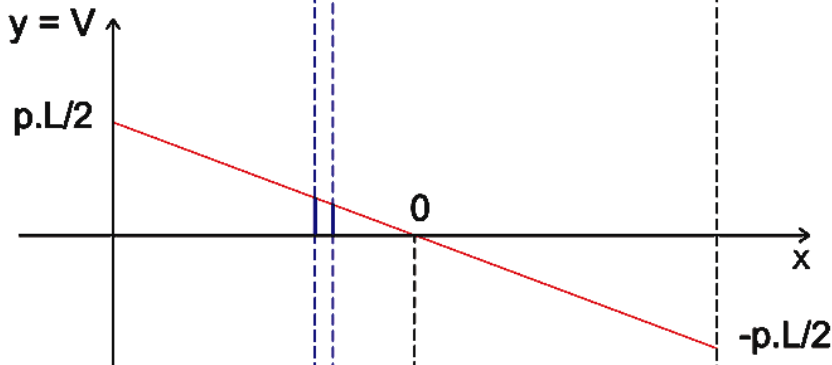
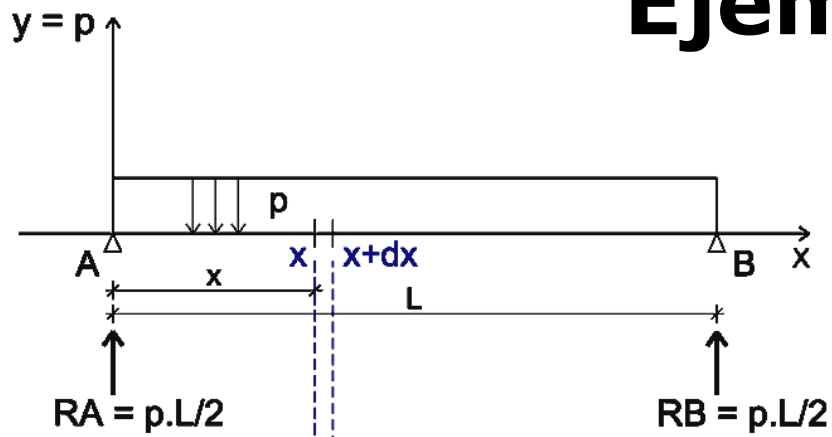
$$M'(x) = p.L / 2 - p.x$$

$$M'(x) = p.L / 2 - p.x = V(x)$$

$$V'(x) = -p$$

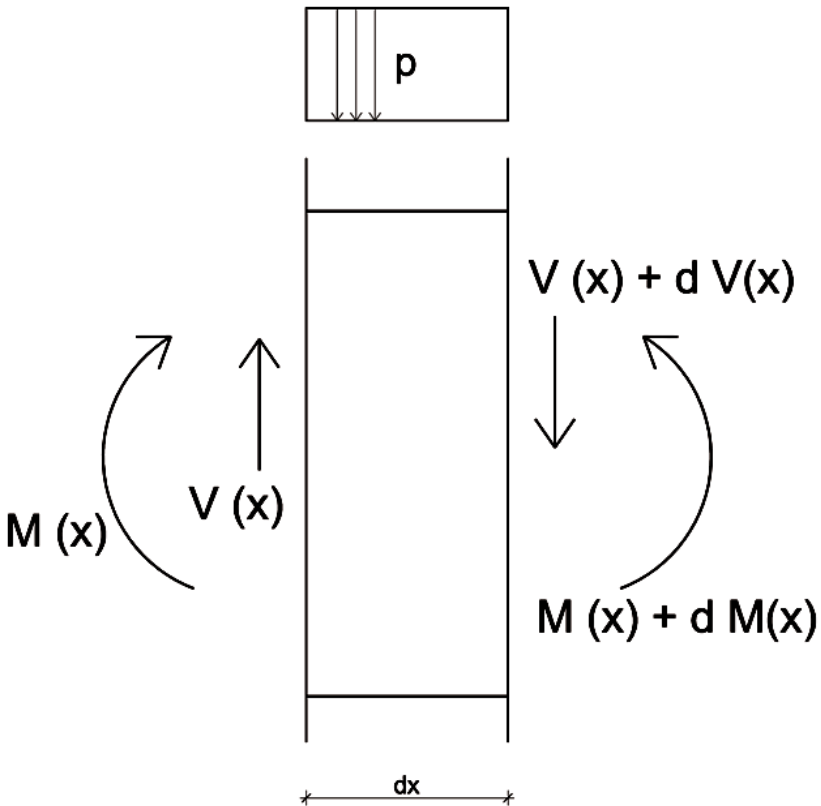
$$M''(x) = V'(x) = -p$$

Ejemplo: viga



Dovela de ancho diferencial

Ejemplo: viga



Equilibrio de la dovela:

$$\sum F_v = 0$$

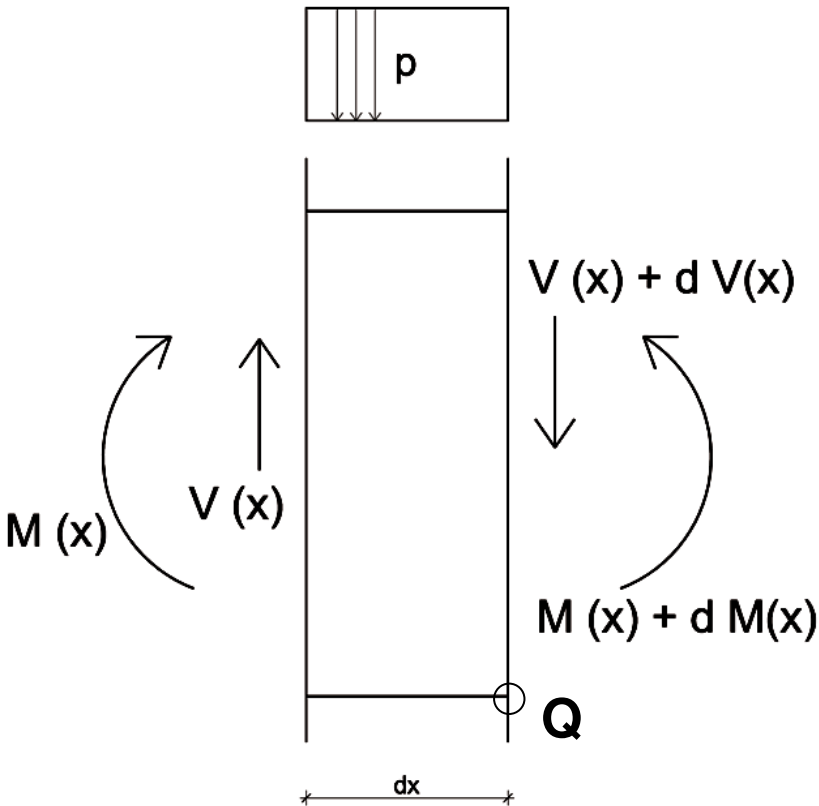
$$V(x) - p \cdot dx - (V(x) + dV(x)) = 0$$

$$V(x) - p \cdot dx - V(x) - dV(x) = 0$$

$$-p \cdot dx = dV(x)$$

$$-p = dV(x) / dx = V'(x)$$

$$\boxed{-p = V'(x)}$$



Equilibrio de la dovela:
 $\sum M_Q = 0$

$$M(x) - (M(x) + dM(x)) + V(x).dx - p.dx.dx/2 = 0$$

$$M(x) - M(x) - dM(x) + V(x).dx - p.dx^2 / 2 = 0$$

$$- dM(x) + V(x).dx = 0$$

$$V(x).dx = dM(x)$$

$$V(x) = dM(x) / dx = M'(x)$$

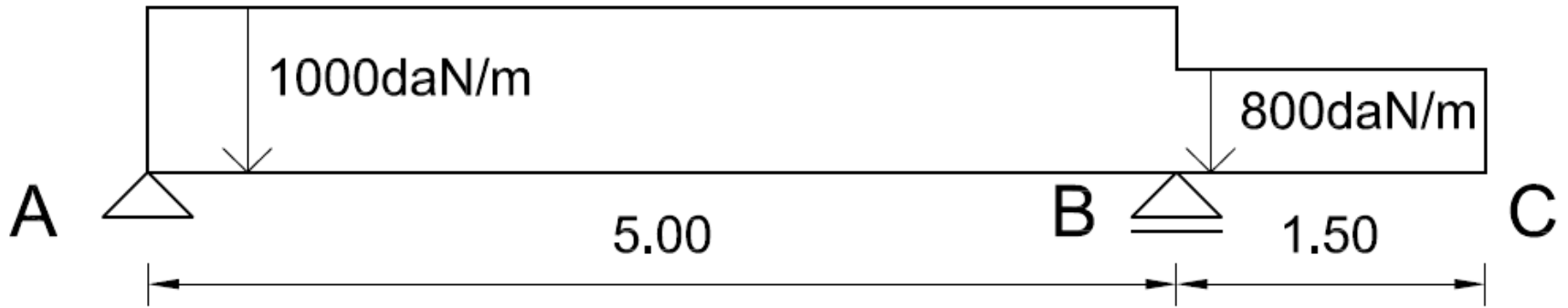
$$V(x) = M'(x)$$

$$-p = V'(x)$$
$$V(x) = M'(x)$$

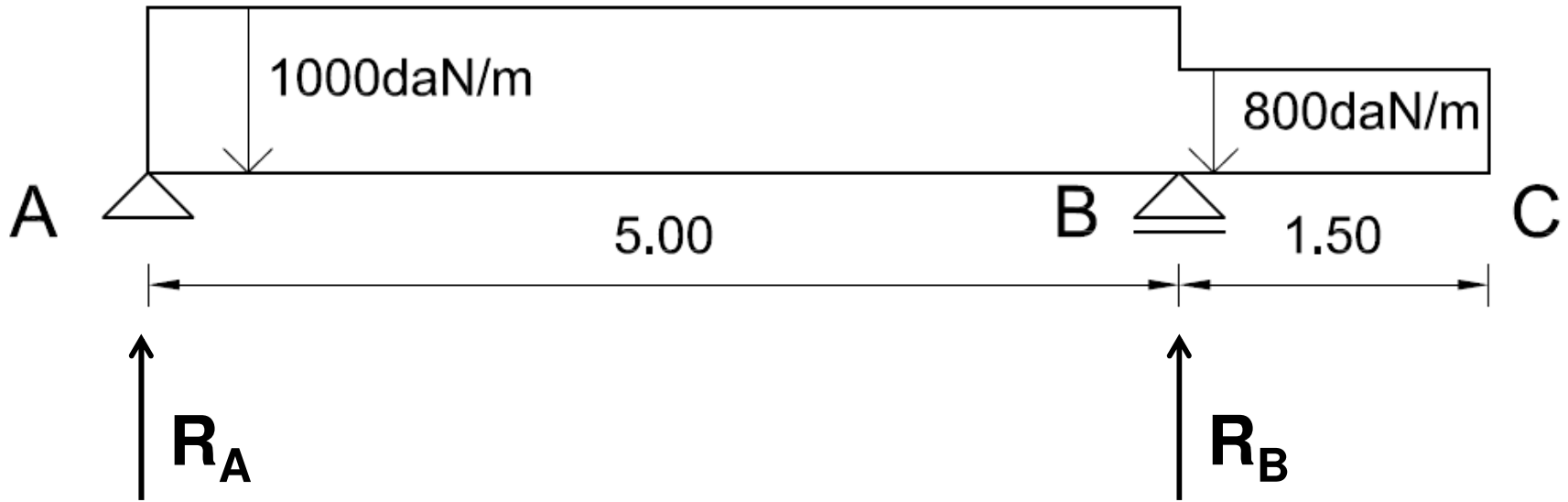
$$M''(x) = V'(x) = -p$$

ecuación fundamental de las vigas rectas

EJEMPLO



EJEMPLO



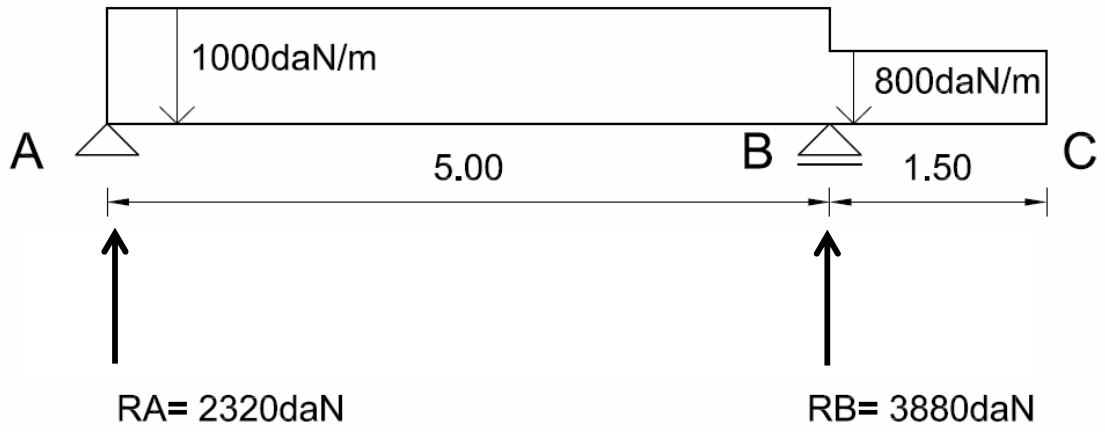
EQUILIBRIO GLOBAL:

$$\sum M_B = 0 \quad -5000 \times 2,5 + 1200 \times 0,75 + R_A \times 5 = 0$$

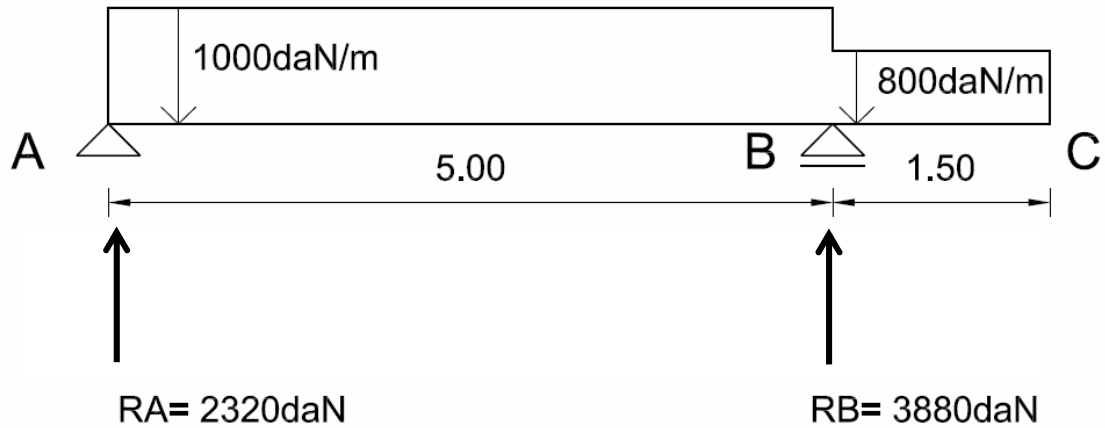
$$R_A = 2320 \text{ daN}$$

$$\sum M_A = 0 \quad 5000 \times 2,5 + 1200 \times 5,75 - R_B \times 5 = 0$$

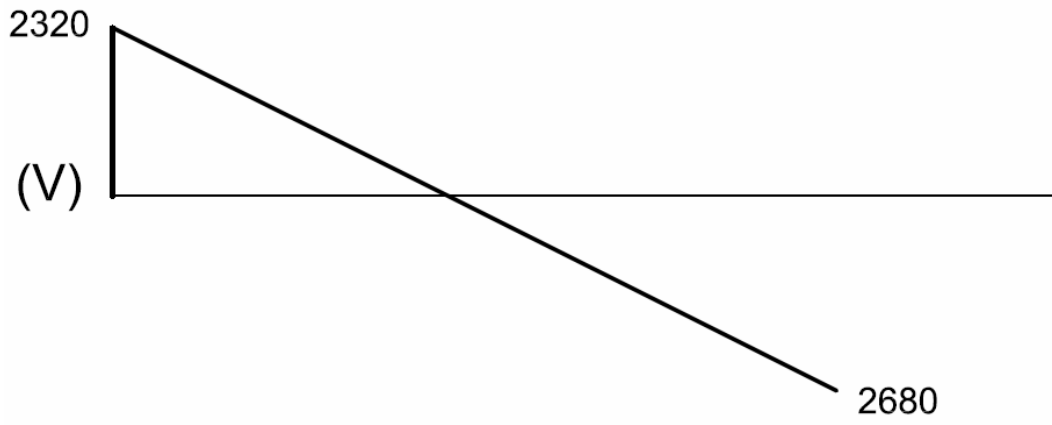
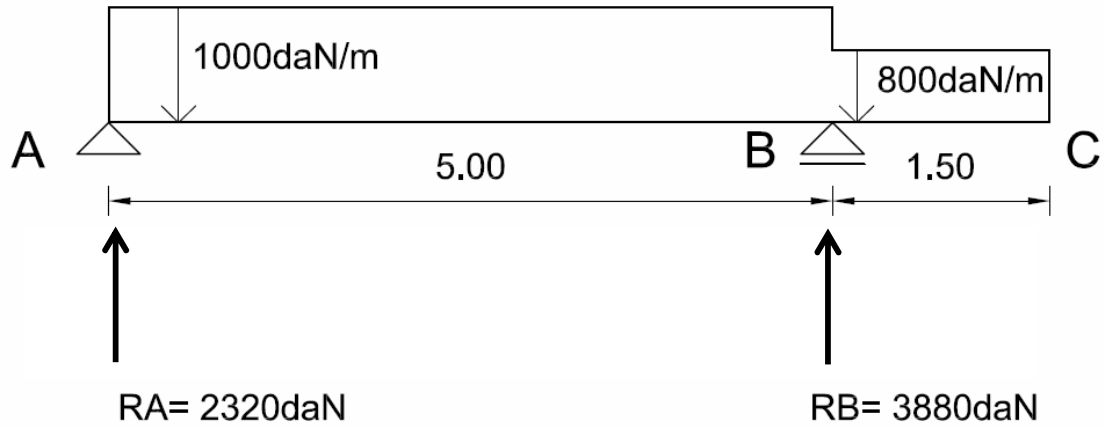
$$R_B = 3880 \text{ daN}$$



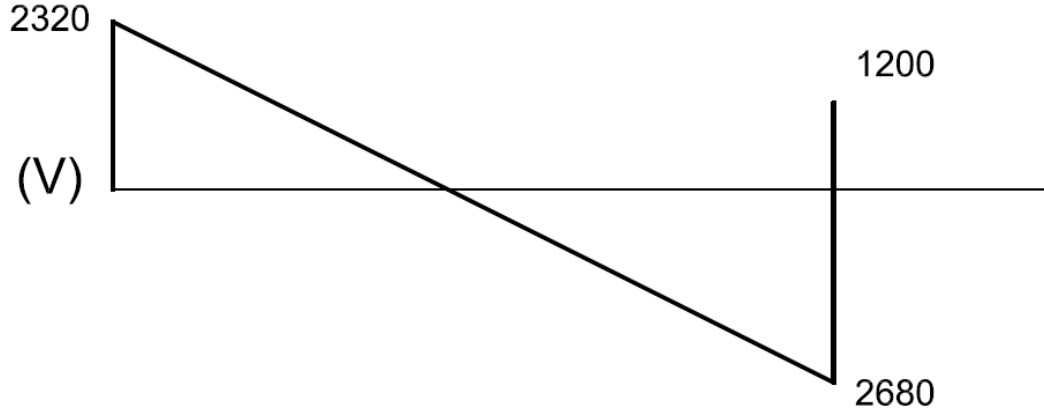
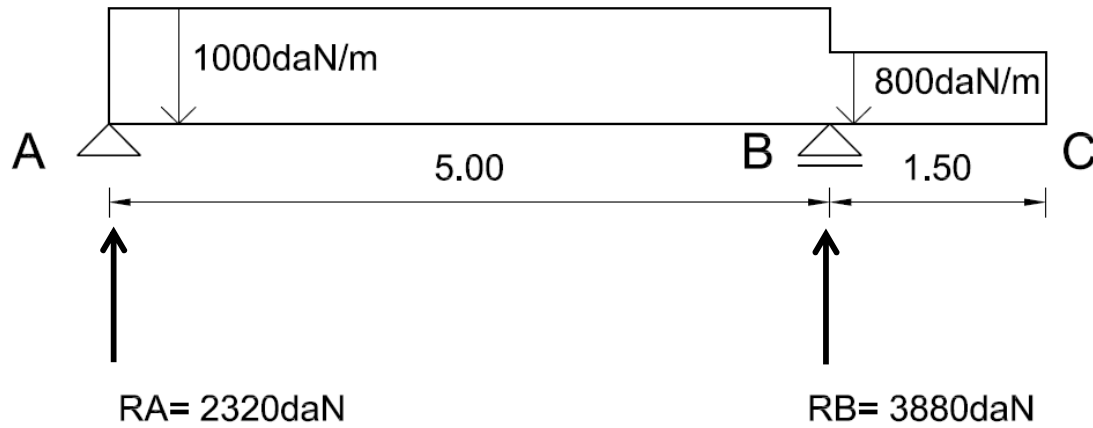
(V) _____



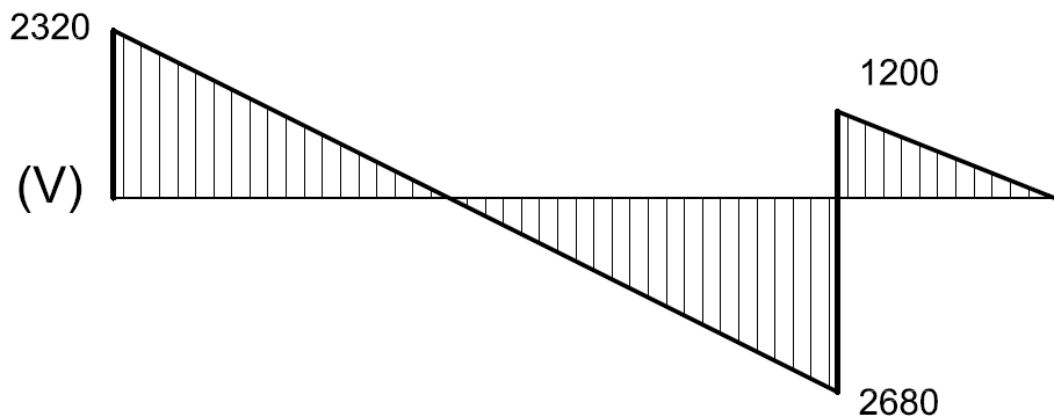
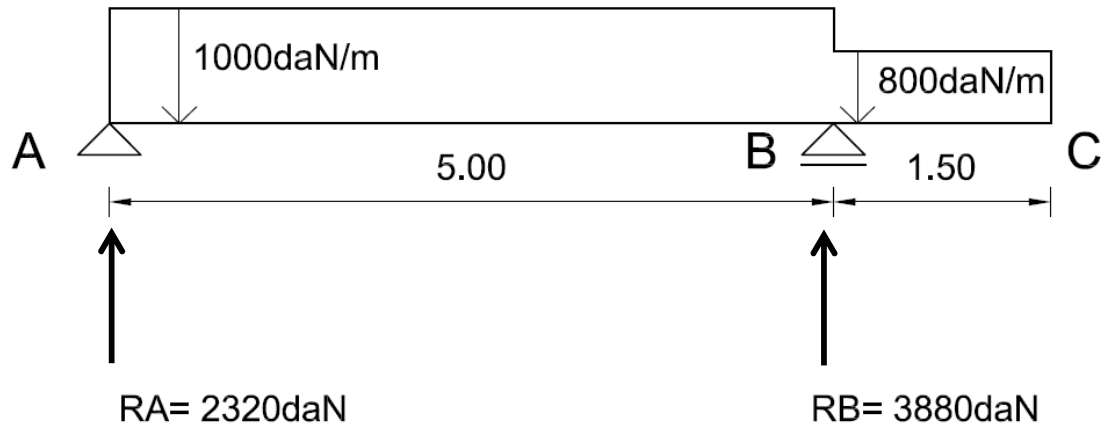
Rizq A = $R_A = 2320 \text{ daN}$



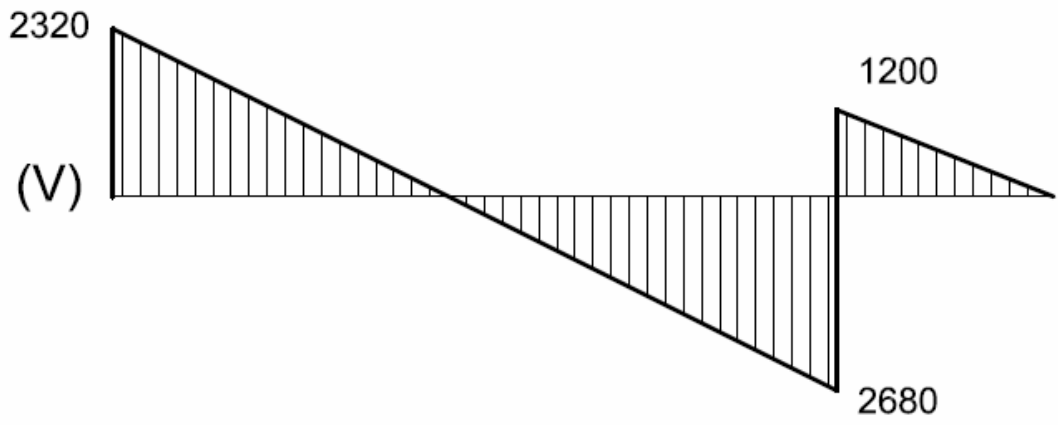
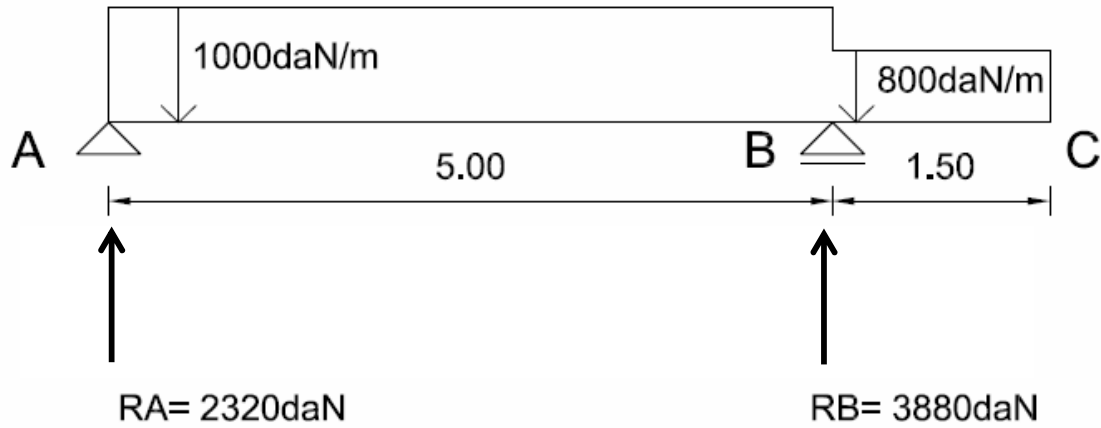
$R_{izq\ A} = R_A = 2320 \text{ daN}$
 $R_{izq\ B1} = R_A - 5000 \text{ daN} = -2680 \text{ daN}$

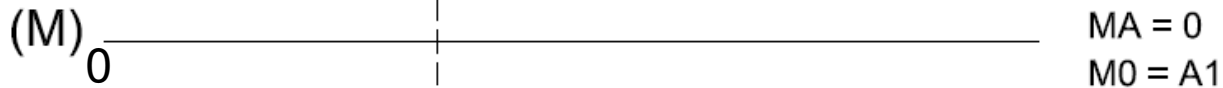
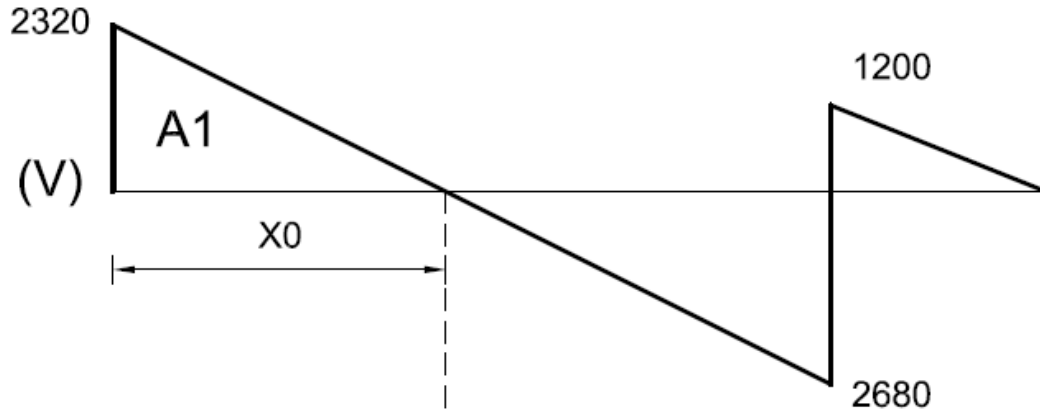
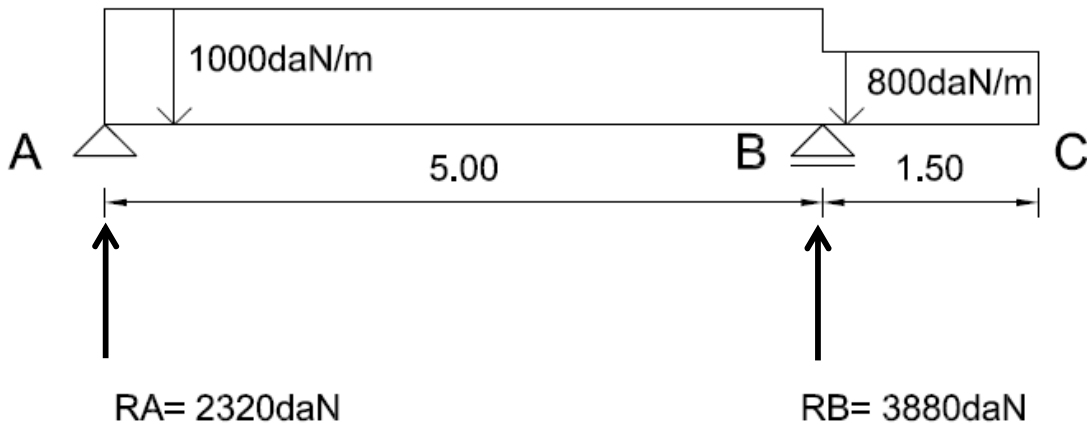


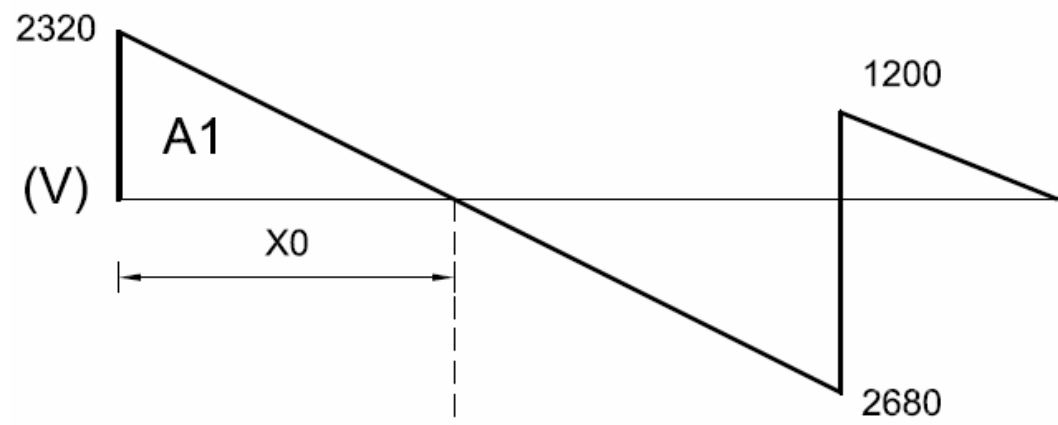
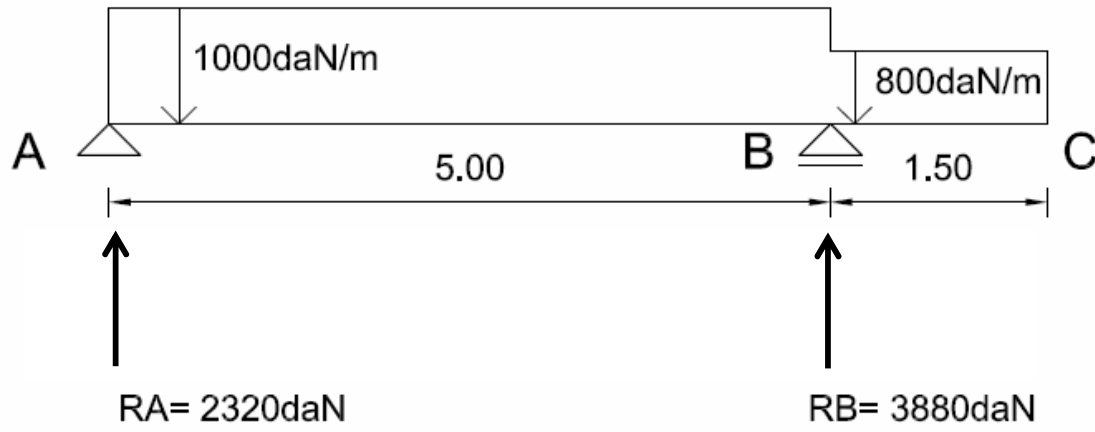
$R_{izq\ A} = R_A = 2320 \text{ daN}$
 $R_{izq\ B1} = R_A - 5000 \text{ daN} = -2680 \text{ daN}$
 $R_{izq\ B2} = R_A - 5000 \text{ daN} + R_B = 1200 \text{ daN}$



$$\begin{aligned}
 \text{Rizq A} &= R_A = 2320 \text{ daN} \\
 \text{Rizq B1} &= R_A - 5000 \text{ daN} = -2680 \text{ daN} \\
 \text{Rizq B2} &= R_A - 5000 \text{ daN} + R_B = 1200 \text{ daN} \\
 \text{Rizq C} &= R_A - 5000 \text{ daN} + R_B - 1200 \text{ daN} = 0
 \end{aligned}$$



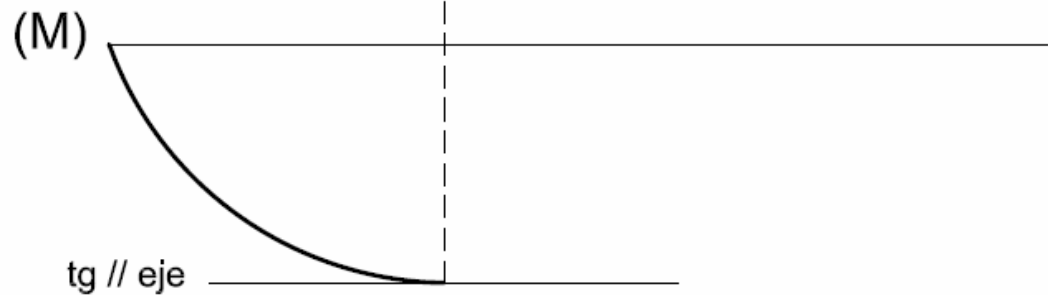




$$x_0 = \frac{V}{p}$$

$$x_0 = \frac{2320 \text{ daN}}{1000 \text{ daN/m}}$$

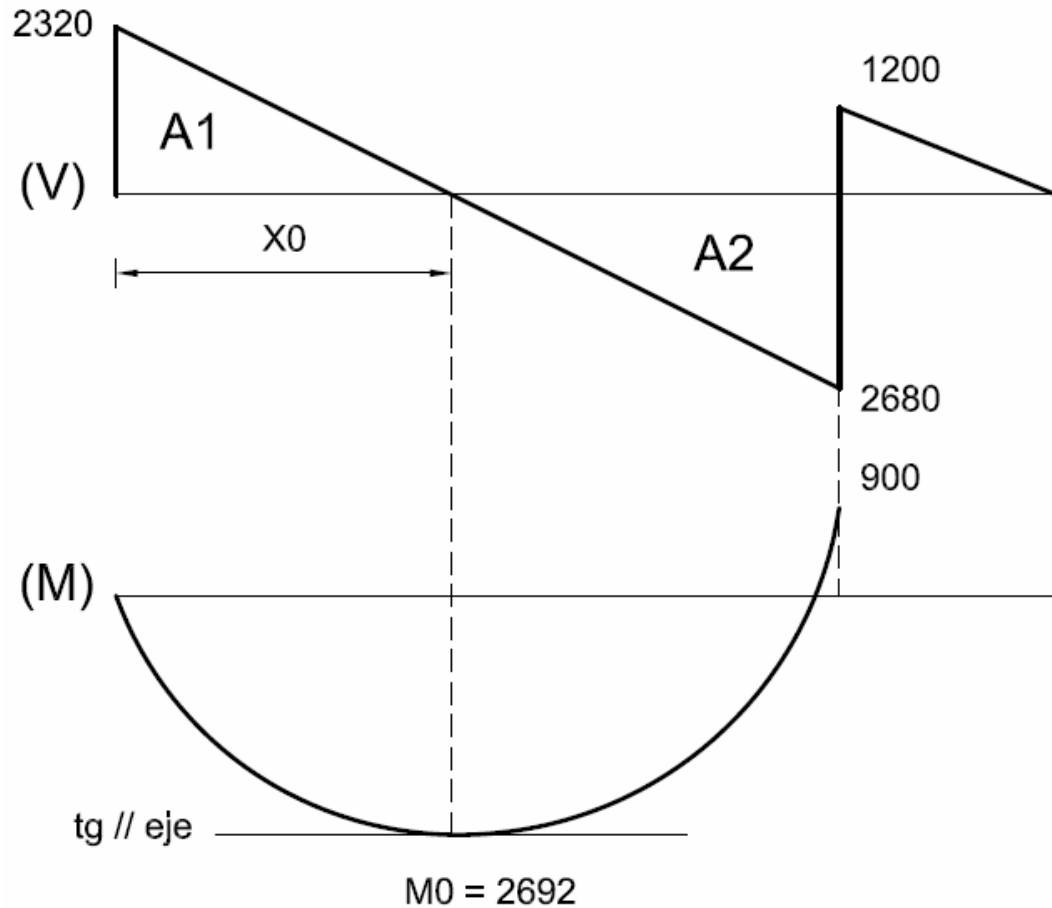
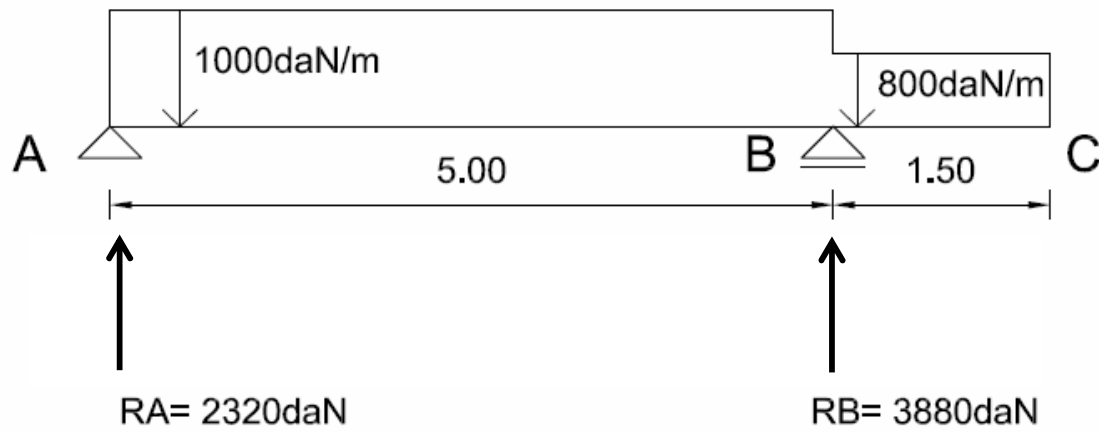
$$x_0 = 2.32 \text{ m}$$



$$M_A = 0$$

$$M_0 = A1 = \frac{2320 \text{ daN} \times 2.32 \text{ m}}{2}$$

$$M_0 = 2692 \text{ daNm}$$



$$x_0 = \frac{V}{p}$$

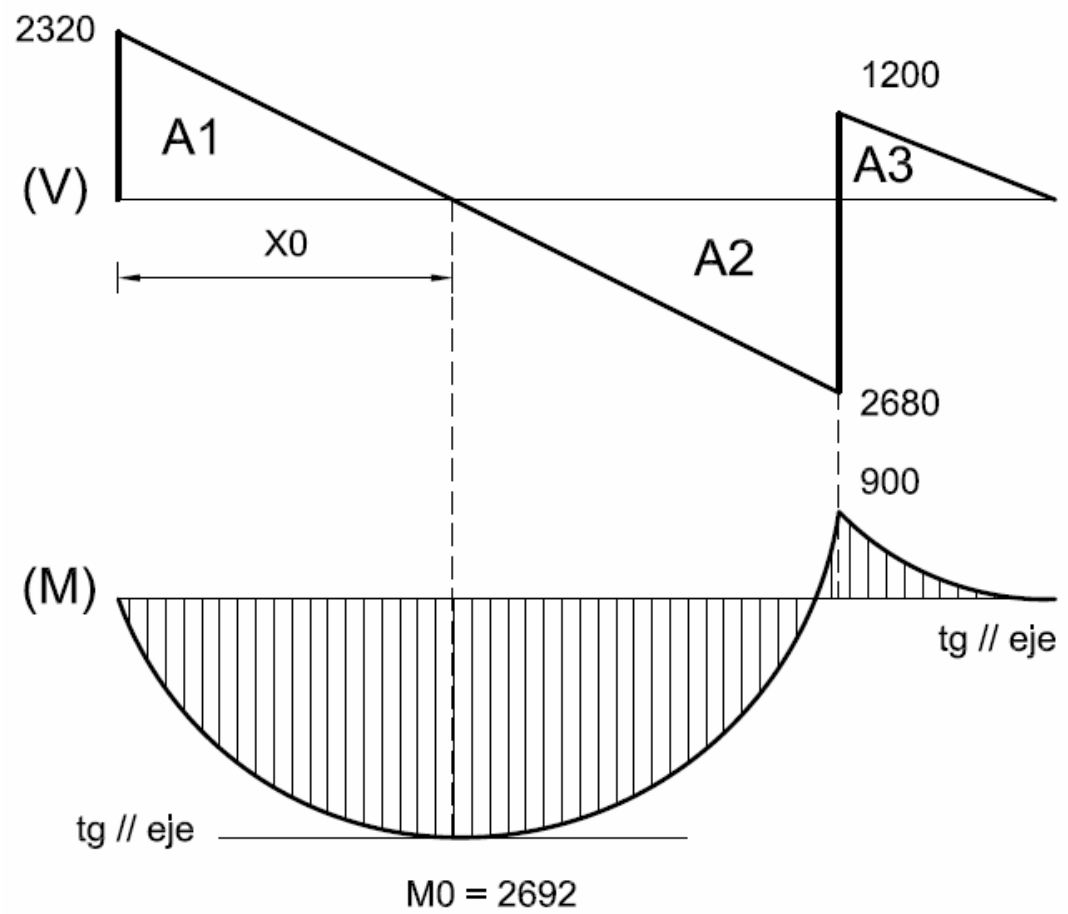
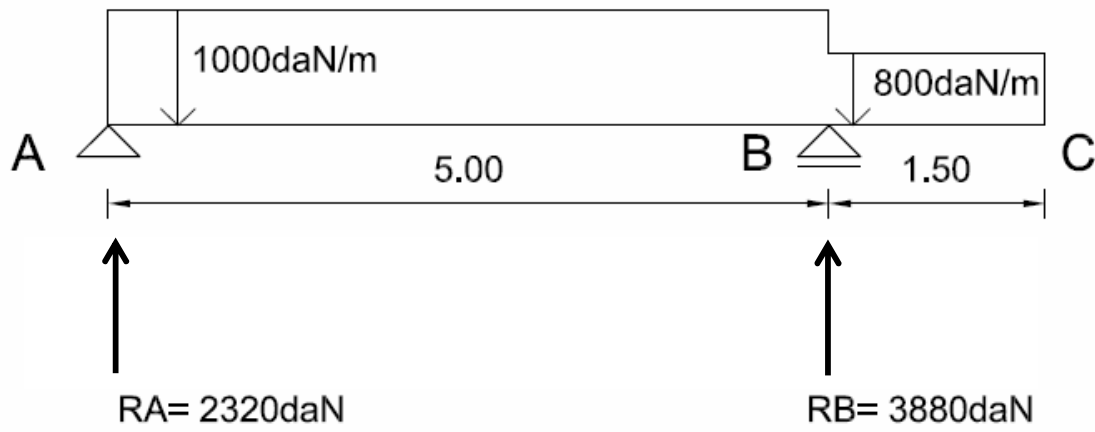
$$x_0 = \frac{2320 \text{ daN}}{1000 \text{ daN/m}}$$

$$x_0 = 2.32 \text{ m}$$

$$M_A = 0$$

$$M_0 = A_1 = 2692 \text{ daNm}$$

$$M_B = A_1 - A_2 = -900 \text{ daNm}$$



$$x_0 = \frac{V}{p}$$

$$x_0 = \frac{2320 \text{ daN}}{1000 \text{ daN/m}}$$

$$x_0 = 2.32 \text{ m}$$

$$M_A = 0$$

$$M_0 = A_1 = 2692 \text{ daNm}$$

$$M_B = A_1 - A_2 = -900 \text{ daNm}$$

$$M_B = A_1 - A_2 + A_3$$

$$M_C = A_1 - A_2 + A_3 = 0$$

