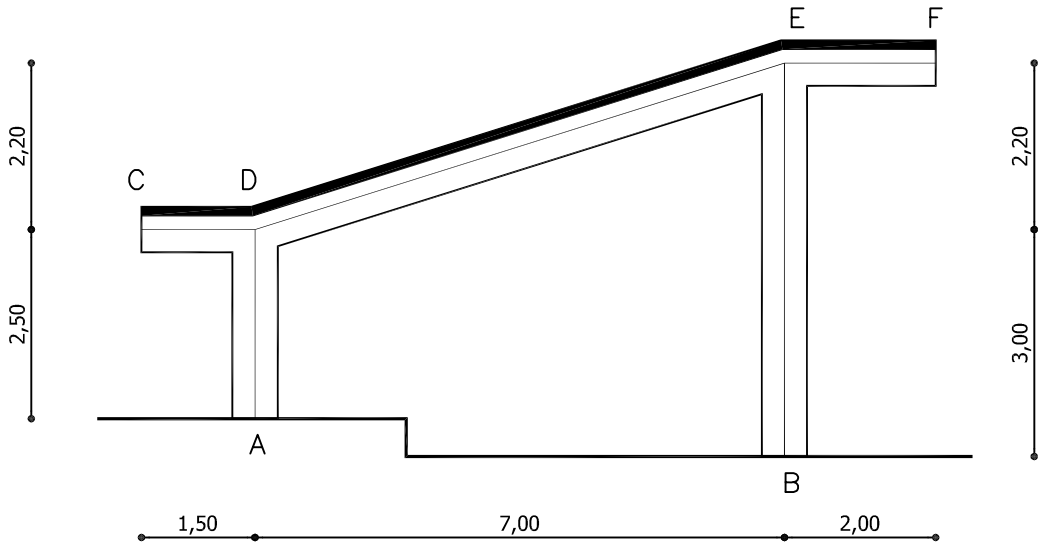
ESTABILIDAD DE LAS CONSTRUCCIONES II

Parte escrita del examen.
13–12–02

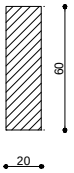
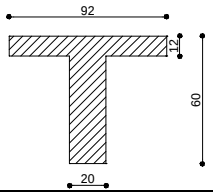
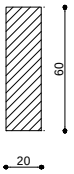
Se plantea la siguiente estructura:



Se trata de una costilla intermedia de hormigón armado en la que apoyan losas macizas de 12 cm de espesor que descargan sobre los tramos CD, DE y EF una carga vertical de 1600daN/m de tramo. La costilla tiene 20 cm de espesor y 60 cm de altura en todos los tramos.

Estudiándola por Método de Cross se pide:
a) Diagrama de Solicitaciones de todos los tramos
b) Reacciones en los apoyos.
c) Verificación de la sección más comprometida, proponiendo ajustes de sus dimensiones si fuera necesario.

Determinación de los coeficientes α y β, y las rigideces de los tramos:

TRAMO	$Li(m)$	Tipo de Sección	Im_i	$I_r = \frac{\mathrm{Im}_i}{I_{MIN}}$	$\chi = \frac{I_r}{L_i}$	α_i	$\alpha_i \cdot \chi$	β
AD	2,50		$\frac{20.60^3}{12}$	1	0,4	1	0,4	0,5
DE	7,34		$\frac{0{,}4.92.60^3}{12}$	1,84	0,25	1	0,25	0,5
EB	5,20		$\frac{20.60^3}{12}$	1	0,19	1	0,19	0,5

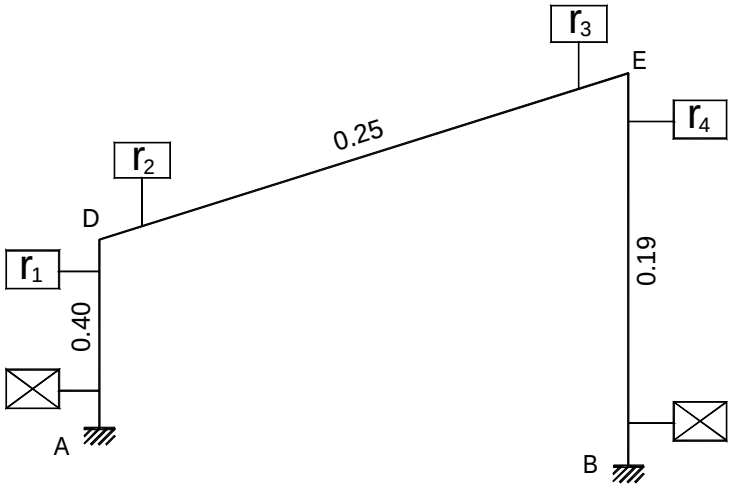
Barra DE:

$$b_e = 6.h_f + b_w = 6.12 + 20 = 92cm$$

Cálculo de la inercia por medio de la Tabla III-4:

$$\xi' = \frac{h_f}{h} = \frac{12}{60} = 0{,}2$$
$$\xi = \frac{b_w}{b_e} = \frac{20}{92} = 0{,}22 \qquad \Rightarrow \psi = 0{,}40 \Rightarrow \quad I_m = \frac{\psi.b_e.h^3}{12} = \frac{0{,}40.92.60^3}{12}$$

Determinación de los Coeficientes de Repartición:



Nudo D:

$$\sum \alpha_i . \chi_i = 0,4 + 0,25 = 0,65$$

$$r_1 = \frac{0,4}{0,65} = 0,62$$

$$r_2 = \frac{0,25}{0,65} = 0,38$$

Nudo E:

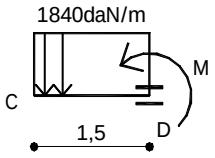
$$\sum \alpha_i . \chi_i = 0,25 + 0,19 = 0,44$$

$$r_3 = \frac{0,25}{0,44} = 0,57$$

$$r_4 = \frac{0,19}{0,44} = 0,43$$

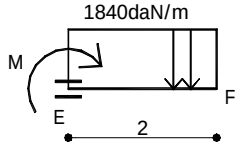
Determinación de Cargas:

Ménsula CD:



$$M = \frac{1840 \times 1,5^2}{2} = 2070 daN.m$$

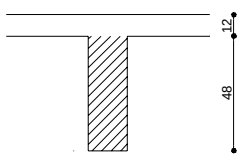
Ménsula EF:



$$M = \frac{1840 \times 2^2}{2} = 3680 daN.m$$

Determinación de los Momentos Freno:

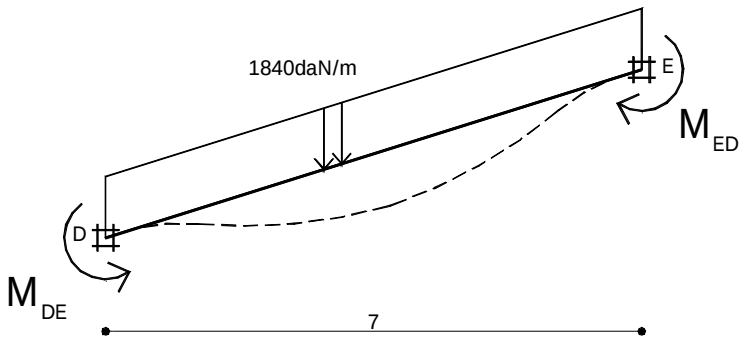
Barra DE:



$p.p. = 0{,}48 \times 0{,}20 \times 2500 = 240 daN/m$

Descarga de la losa = $1600 daN/m$

Total = $1840 daN/m$

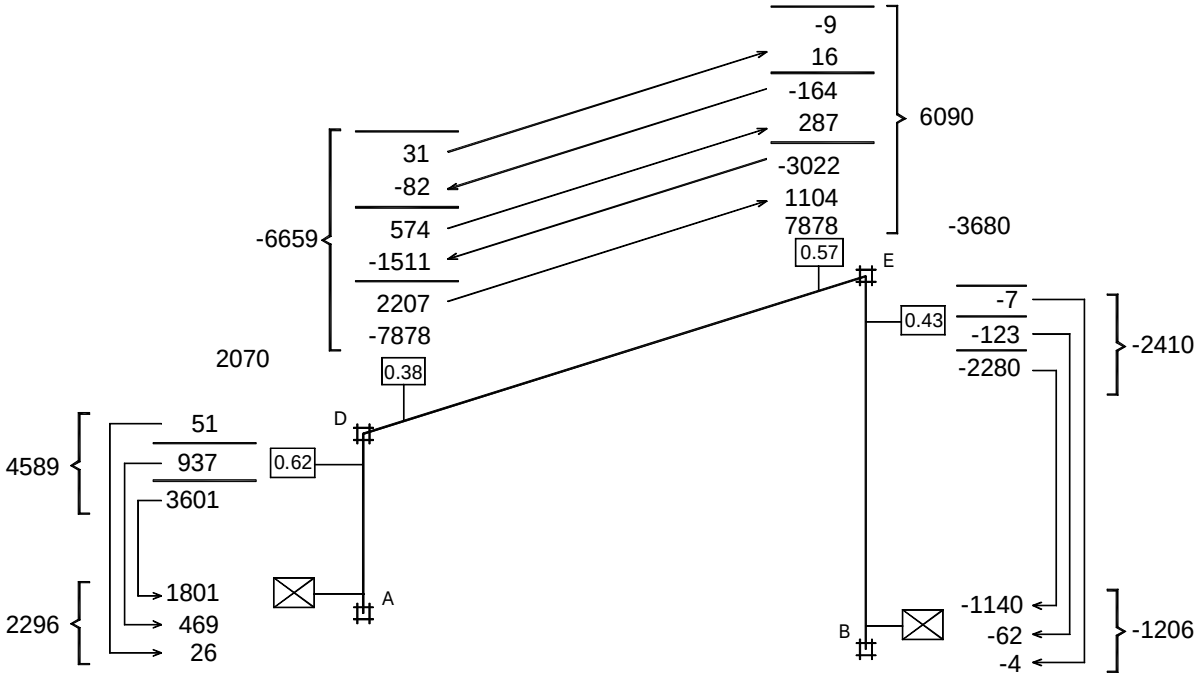


$$M = \frac{p_i.L_i.L_H}{12} = \frac{1840.7,34.7}{12} = 7878 daN.m$$

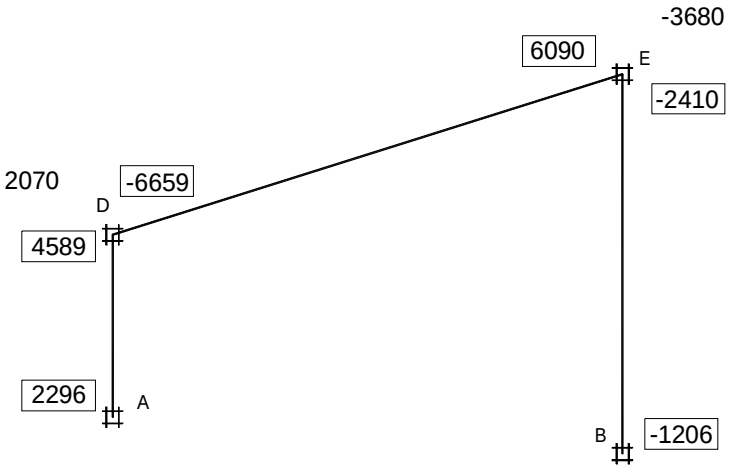
$$M_{DE} = -7878 daN.m$$

$$M_{ED} = 7878 daN.m$$

Artificio del 1º Cross:

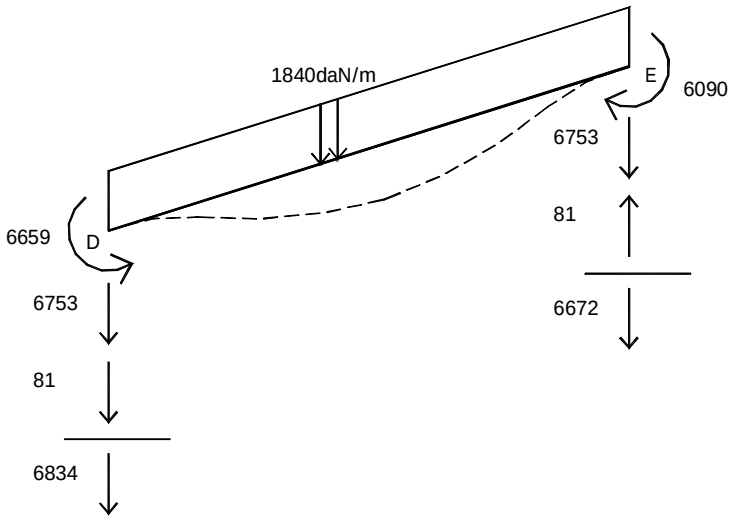


Resumen:



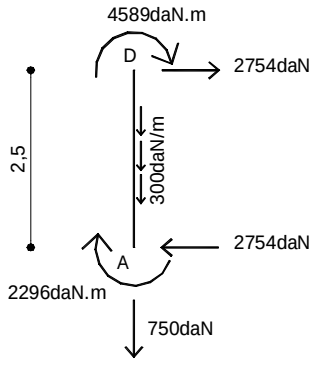
Descargas en los nodos:

Barra DE:



$$\frac{1840 \times 7{,}34}{2} = 6753 daN \qquad \frac{6659 - 6090}{7} = 81 daN$$

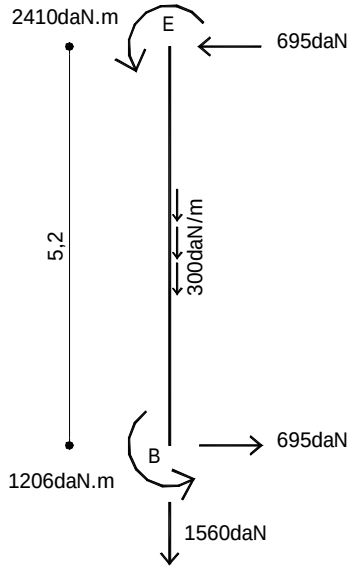
Barra DA:



$$p.p. = 0{,}6 \times 0{,}20 \times 2500 = 300 daN/m$$
$$300 \times 2{,}50 = 750 daN$$

$$\frac{4589 + 2296}{2{,}50} = 2754 daN$$

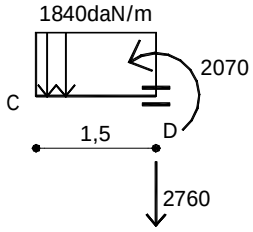
Barra EB:



$$\frac{2410+1206}{5,20}=695daN$$

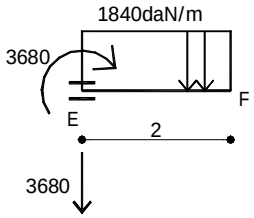
$$300\times 5,20=1560daN$$

Ménsula CD:



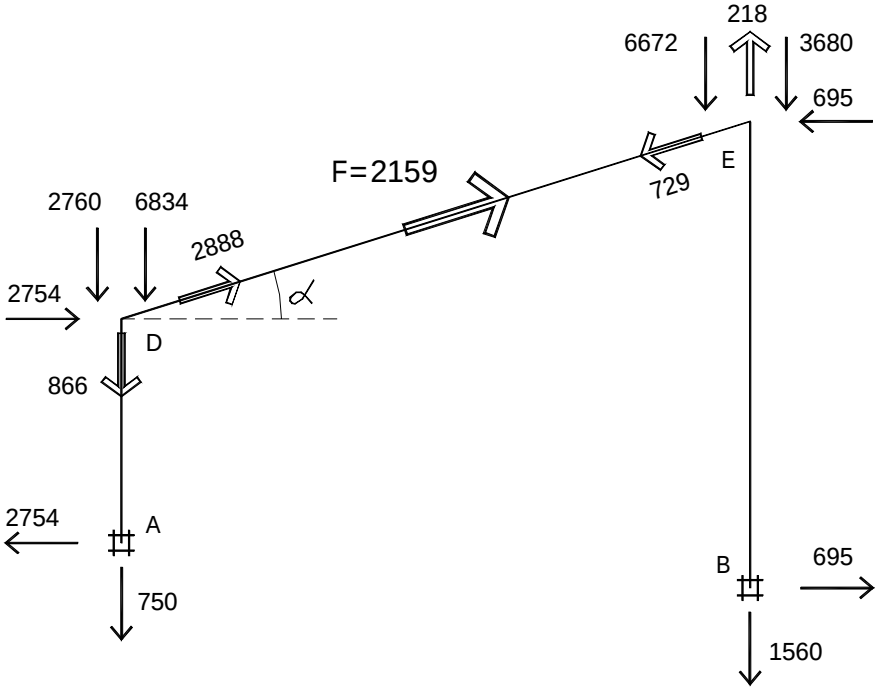
$$F=1840\times 1,50=2760daN$$

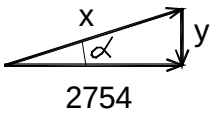
Ménsula EF:



$$F=1840\times 2=3680daN$$

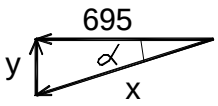
Descargas totales de la estructura y descomposición según caminos materiales:





$$tg\alpha = \frac{y}{2754} = \frac{2,2}{7} \Rightarrow y = \frac{2754.2,2}{7} = 866daN$$

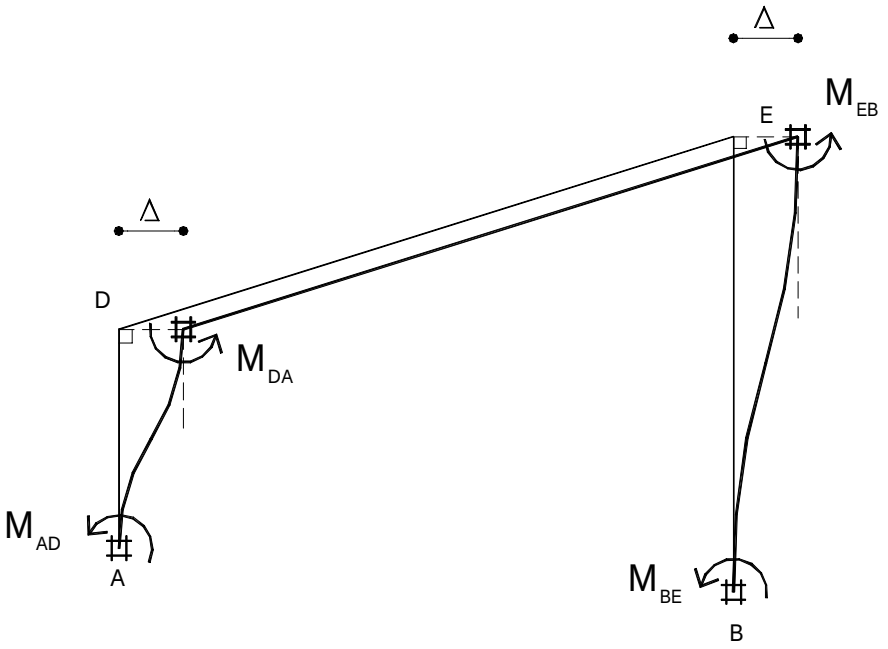
$$\cos\alpha = \frac{2754}{x} = \frac{7}{7,34} \Rightarrow x = \frac{2754.7,34}{7} = 2888daN$$



$$y = \frac{695.2,2}{7} = 218daN$$

$$x = \frac{695.7,34}{7} = 729daN$$

Trazado de la deformada:

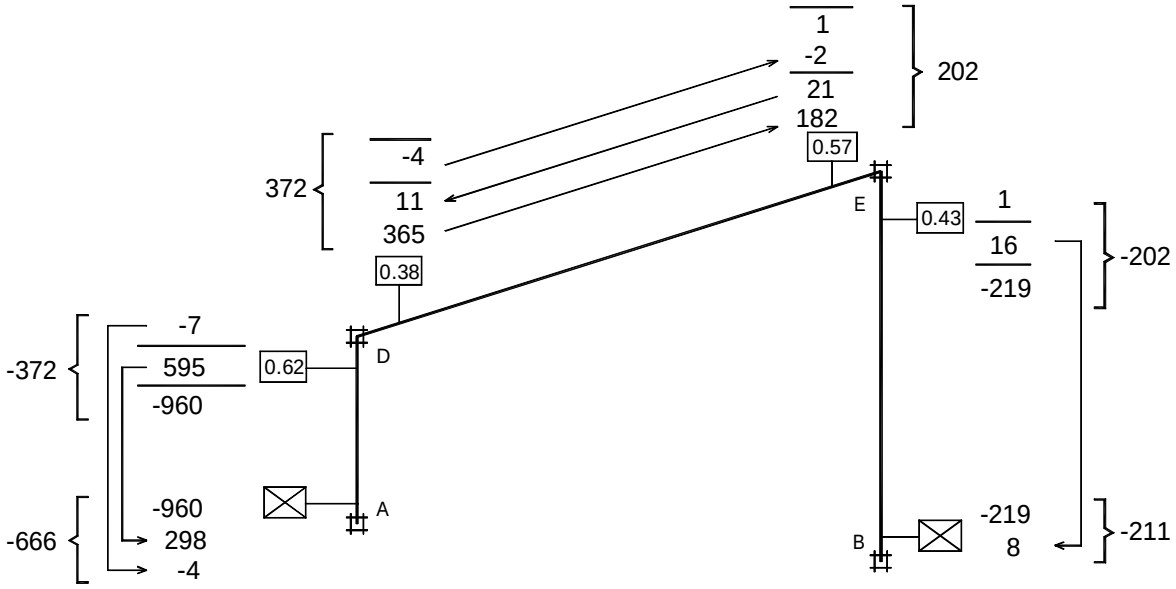


Se toma $\Delta = 1000$

$$M_{AD} = M_{DA} = \frac{6.\chi.\Delta}{L} = \frac{6.0,4.1000}{2,5} = 960daN.m$$

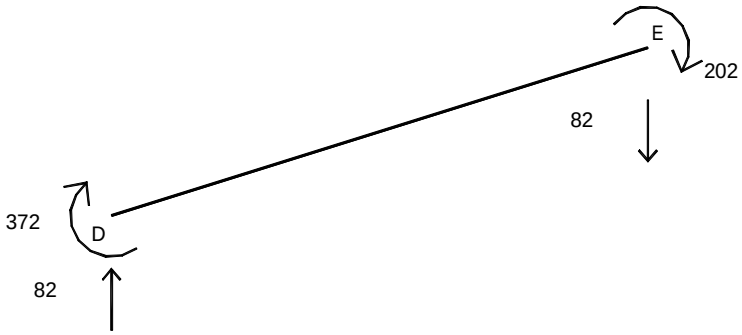
$$M_{EB} = M_{BE} = \frac{6.\chi.\Delta}{L} = \frac{6.0,19.1000}{5,2} = 219daN.m$$

Artificio del 2º Cross:



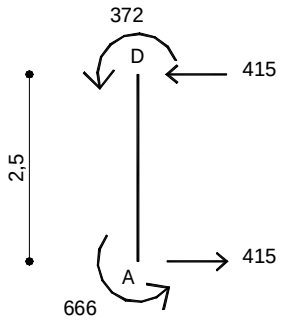
Descargas en los nudos usando solamente los momentos del 2º Cross:

Barra DE:



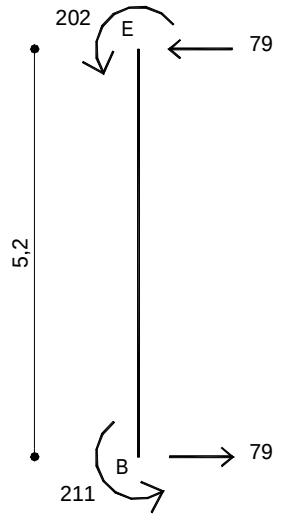
$$\frac{202+372}{7} = 82daN$$

Barra AD:



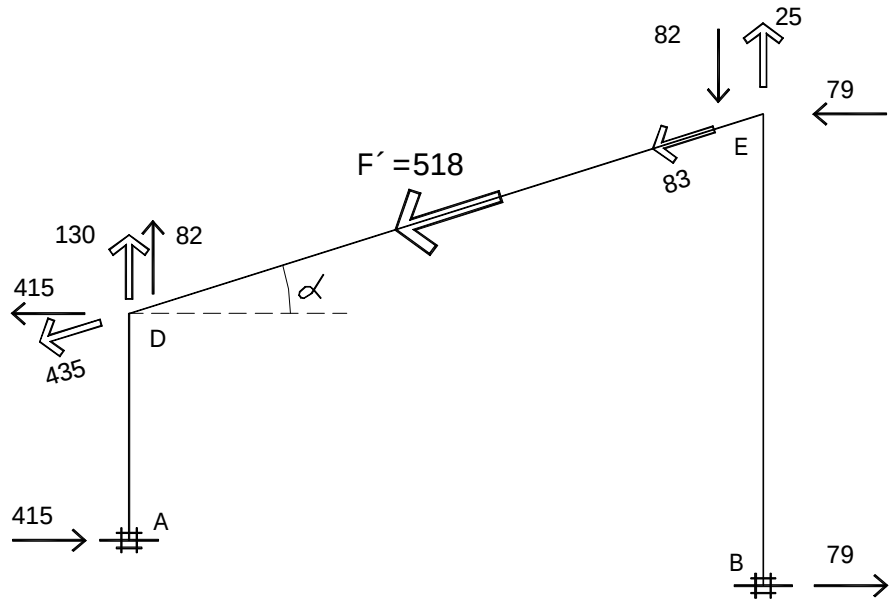
$$\frac{666+372}{2,5} = 415daN$$

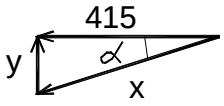
Barra EB:



$$\frac{202+211}{5,2}=79daN$$

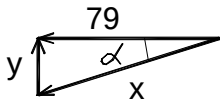
Descomposición según caminos materiales:





$$y=\frac{415.2,2}{7}=130daN$$

$$x=\frac{415.7,34}{7}=435daN$$



$$y=\frac{79.2,2}{7}=25daN$$

$$x=\frac{79.7,34}{7}=83daN$$

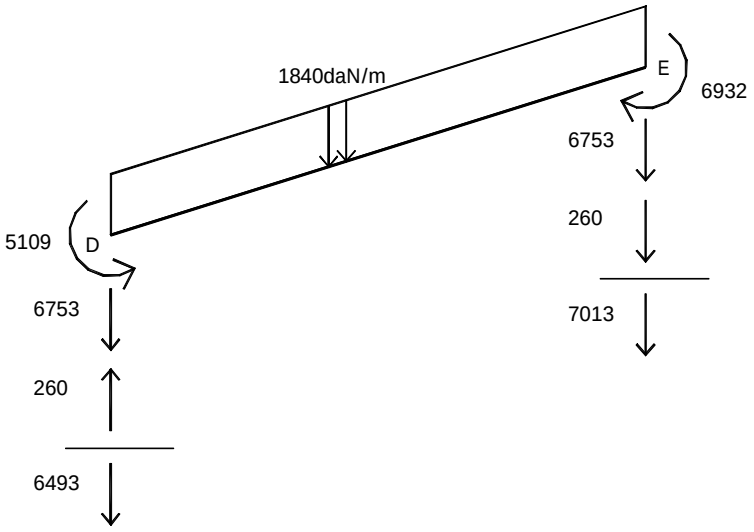
Momentos Finales:

$$\alpha = \frac{F}{F'} = \frac{2159}{518} = 4{,}168$$

	$Mom.1^{o}\,Cross$	$\alpha \times Mom.2^{o}\,Cross$	$Mom.Finales$
M_{AD}	2296	-2776	-480
M_{DA}	4589	-1550	3039
M_{DE}	-6659	1550	-5109
M_{ED}	6090	842	6932
M_{EB}	-2410	-842	-3252
M_{BE}	-1206	-875	-2081

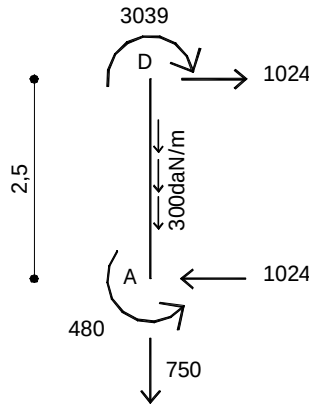
Descargas Finales en los nodos:

Barra DE:



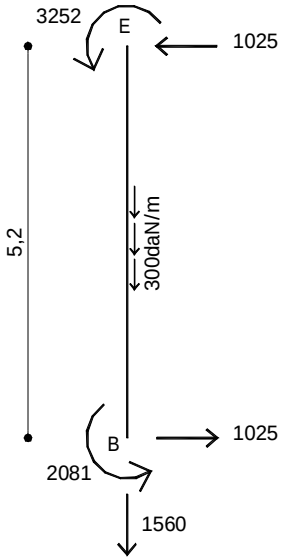
$$\frac{6932 - 5109}{7} = 260daN$$

Barra AD:

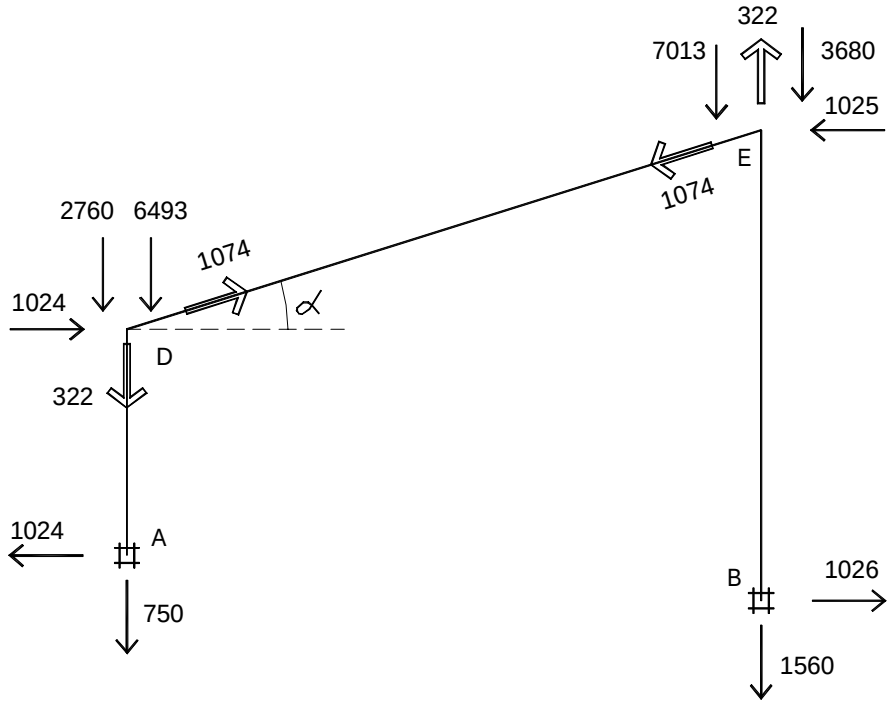


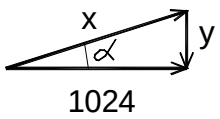
$$\frac{3039 - 480}{2{,}5} = 1024daN$$

Barra EB:



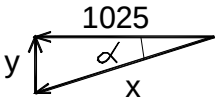
$$\frac{3252+2081}{5,2}=1025daN$$





$$y=\frac{1024.2,2}{7}=322daN$$

$$x=\frac{1024.7,34}{7}=1074daN$$

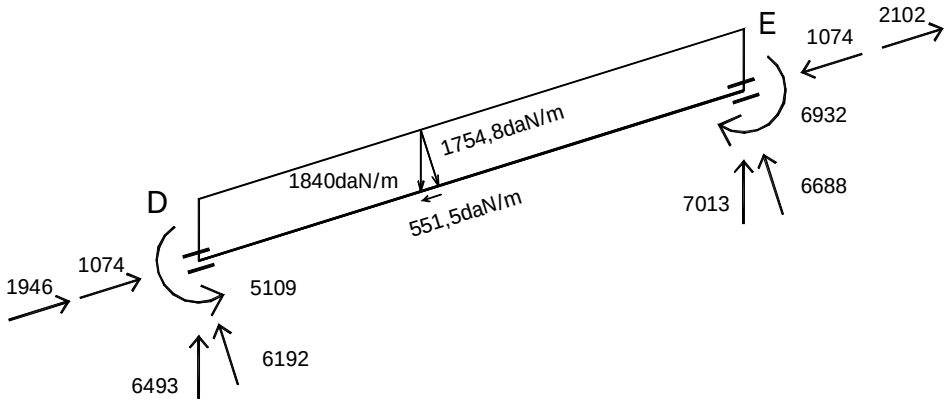


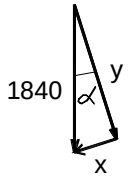
$$y=\frac{1025.2,2}{7}=322daN$$

$$x=\frac{1025.7,34}{7}=1074daN$$

Equilibrio de cada tramo y Diagrama de Solicitaciones:

Barra DE:



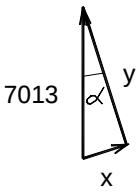


$$sen\alpha = \frac{x}{1840} = \frac{2,20}{7,34} \Rightarrow x = \frac{1840.2,20}{7,34} = 551,5daN$$

$$\cos\alpha = \frac{y}{1840} = \frac{7}{7,34} \Rightarrow y = \frac{1840.7}{7,34} = 1754,8daN$$

$$x = \frac{6493.2,20}{7,34} = 1946daN$$

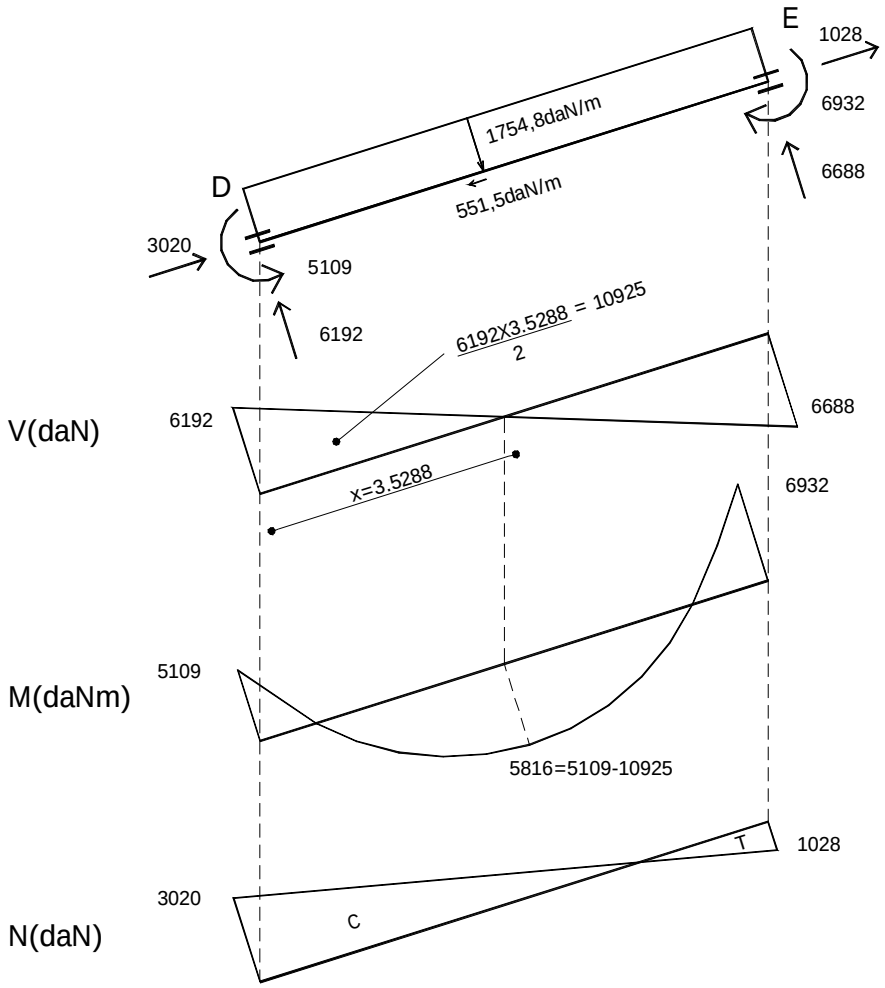
$$y = \frac{6493.7}{7,34} = 6192daN$$



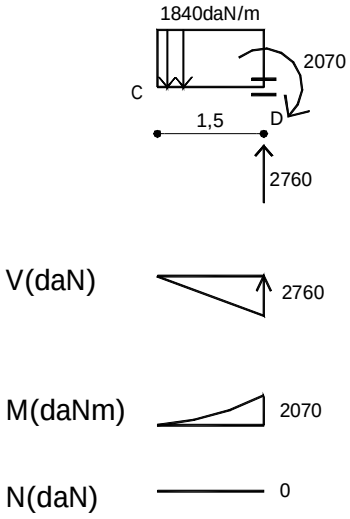
$$x = \frac{7013.2,20}{7,34} = 2102daN$$

$$y = \frac{7013.7}{7,34} = 6688daN$$

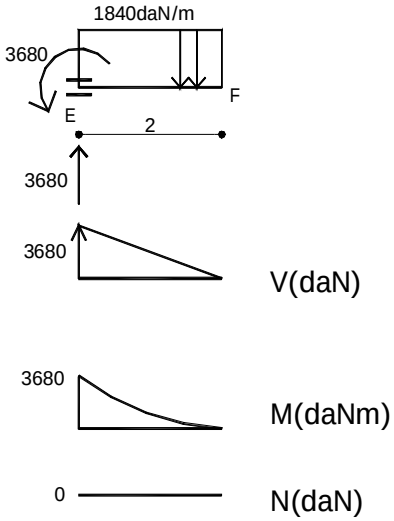
$$x_0 = \frac{6493}{1840} = 3,5288m$$


E
1028
1754,8daN/m
6932
6688
D
551,5daN/m
3020
5109
6192
6192X3.5288 = 10925
2
V(daN)
6192
6688
x=3.5288
6932
M(daNm)
5109
5816=5109-10925
T
1028
3020
C
N(daN)

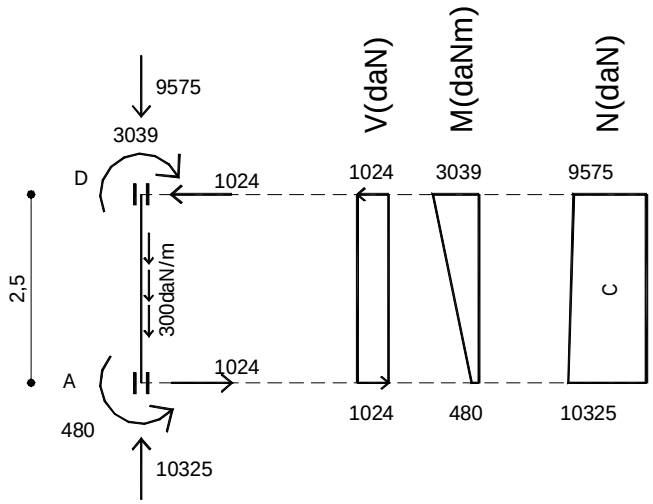
Ménsula CD:


1840daN/m
2070
C
1,5
D
2760
V(daN)
2760
M(daNm)
2070
N(daN)
0

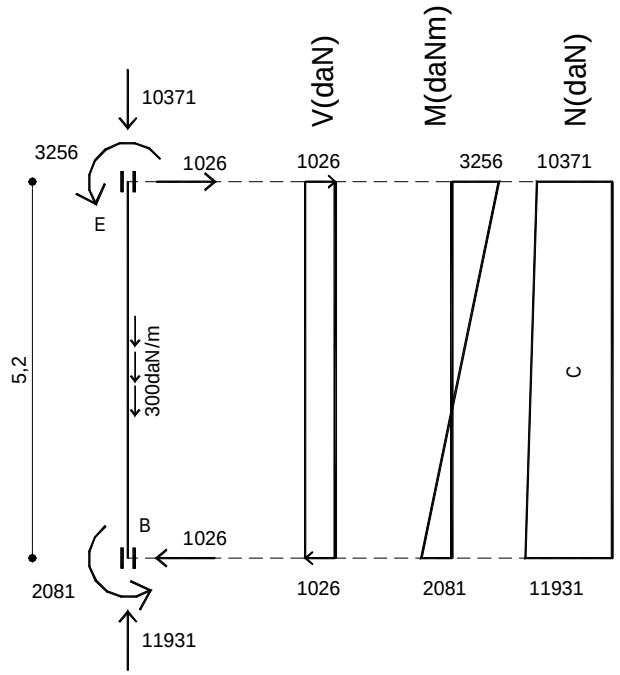
Ménsula EF:



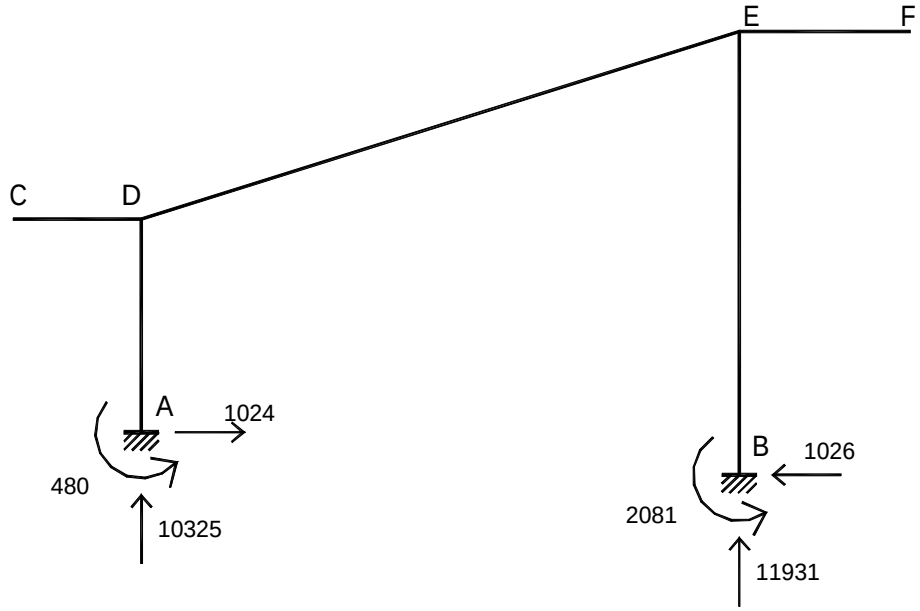
Barra AD:



Barra EB:



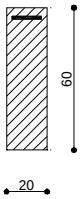
Reacciones en Apoyos:



Verificación de la sección más comprometida:

Barra DE:

Apoyo E



$$M = 6932daN.m \Rightarrow M_d = 11091daN.m$$

$$N_{Tracc.} = 1028daN \Rightarrow N_d = 1645daN$$

$$e = \frac{M_d}{N_d} = \frac{11091}{1645} = 6{,}74m$$

$$\frac{Z_S}{2} = \frac{0{,}54}{2} = 0{,}27m$$

$$\Rightarrow e \rangle \frac{Z_S}{2} \Rightarrow$$ Caso de Gran Excentricidad

$$M_{ad} = M_d - \frac{N_d.Z_s}{2} = 11091 - \frac{1645.0{,}54}{2} = 10647daN.m$$

$$M_{dLIM} = 0{,}332.b.d^2.fcd = 0{,}332.0{,}20.57^2.100 = 21573daN.m$$

$$\Rightarrow M_{ad} \langle M_{dLIM} \Rightarrow$$ Solución Simplemente Armada

$$\mu_{ad} = \frac{M_{ad}}{b.d^2.fcd} = \frac{1064700}{20.57^2.100} = 0{,}164 \Rightarrow \omega = 0{,}184$$

$$A_{S1} = \omega.b.d.\frac{fcd}{fyd} + \frac{N_d}{fyd} = 0{,}184.20.57.\frac{100}{3650} + \frac{1645}{3650} = 6{,}20cm^2$$

Viabilidad:

$$\rho = \frac{A_{S1}}{b.d} = \frac{6{,}20}{20.57} = 0{,}0005 \langle 0{,}018 \Rightarrow Viable$$