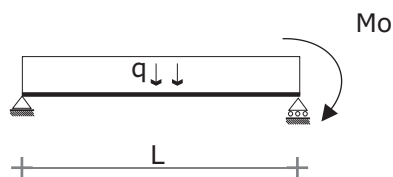
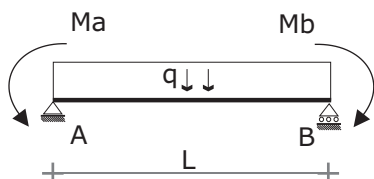


FLECHAS MÁXIMAS DE VIGAS

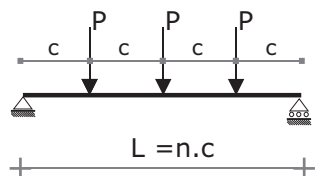


$$* \quad Z(c) = Z(\max) = Z(\text{tramo}) - \frac{1}{16} \cdot \frac{ML^2}{EI}$$



$$* \quad Z(c) = Z(\max) = Z(\text{tramo}) - \frac{1}{16} \cdot \frac{L^2}{EI} \cdot (Ma + Mb)$$

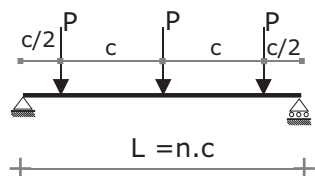
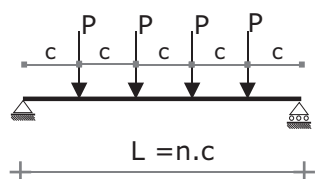
*Si bien la flecha máxima producida por el momento de apoyo y la flecha máxima producida por la carga del tramo no coinciden en el mismo punto, se considera el punto en el centro del tramo y se descuenta el valor de la flecha producida por el momento de apoyo en ese punto, que es un poco menor que la máxima.



n = n° de espacios
n = par / impar

$$Z(\max) = \frac{PL^3}{8EI} \times \frac{1}{\beta}$$

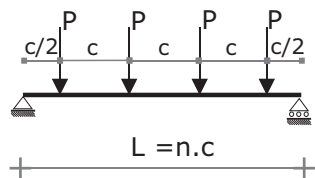
n	2	3	4	5	6	7
β	6	3.52	2.53	1.98	1.64	1.39



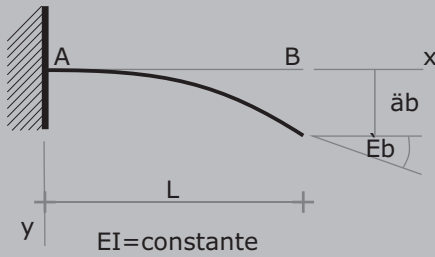
n = n° de espacios
n = par / impar

$$Z(\max) = \frac{PL^3}{8EI} \times \frac{1}{\beta}$$

n	1	2	3	4	5	6	7
β	6	4.36	3.06	2.38	1.89	1.58	1.36



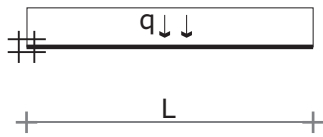
FLECHAS Y PENDIENTES DE MÉNSULAS



$E'(x) = dz/dx =$ pendiente de la curva elástica
 $Z(x) =$ flecha (o deflexión en la dirección y)

$E'(b) = E'(L) =$ pendiente (o ángulo) en el extremo derecho de la viga.

$äb = Z(x) =$ flecha en el extremo derecho de la viga.

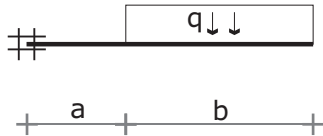


$$E'(x) = \frac{qx}{6EI} (3L^2 - 3Lx + x^2)$$

$$E'(b) = \frac{qL^3}{6EI}$$

$$Z(x) = \frac{qx^2}{24EI} (6L^2 - 4Lx + x^2)$$

$$Z(\max) = \frac{qL^4}{8EI}$$



$$E'(x) = \frac{qbx}{2EI} (L + a - x)$$

$$0 \leq x \leq a$$

$$E'(x) = \frac{qabL}{2EI}$$

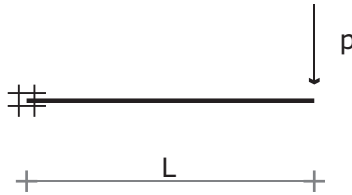
$$x = a$$

$$Z(x) = \frac{qx^2}{12EI} (3bL + 3ab - 2bx)$$

$$Z(x) = \frac{qa^2b}{12EI} (3L + a)$$

$$E'(b) = \frac{q}{6EI} (L^3 - a^3)$$

$$Z(\max) = \frac{q}{24EI} (3L^4 - 4a^3L + a^4)$$

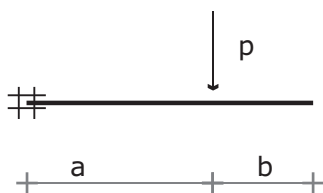


$$E'(x) = \frac{Px}{2EI} (2L - x)$$

$$E'(b) = \frac{PL^2}{2EI}$$

$$Z(x) = \frac{Px^2}{6EI} (3L - x)$$

$$Z(\max) = \frac{PL^3}{3EI}$$



$$E'(x) = \frac{Px}{2EI} (2a - x)$$

$$0 \leq x \leq a$$

$$E'(x) = \frac{Pa^2}{2EI}$$

$$x = a$$

$$Z(x) = \frac{Px^2}{6EI} (3a - x)$$

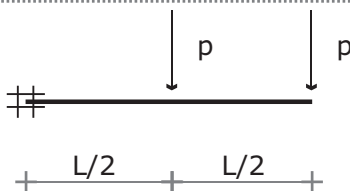
$$Z(x) = \frac{Pa^3}{3EI}$$

$$E'(x) = \frac{Pa^2}{2EI}$$

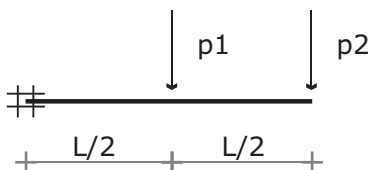
$$a \leq x \leq L$$

$$E'(b) = \frac{Pa^2}{2EI}$$

$$Z(\max) = \frac{Pa^2}{6EI} (3L - a)$$

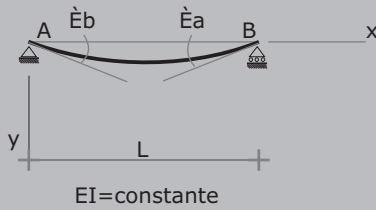


$$Z(\max) = \frac{7PL^3}{16EI}$$



$$Z(\max) = \frac{(p_2)L^3}{3EI} + \frac{5(p_1)L^3}{48EI}$$

FLECHAS Y PENDIENTES DE VIGAS



$\dot{E}(x) = dz/dx =$ pendiente de la curva elástica

$Z(x) =$ flecha (o deflexión en la dirección y)

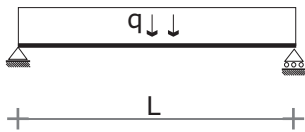
$Z(c) = Z(\frac{L}{2}) =$ flecha en el centro de la viga

$x_1 =$ distancia desde A hasta el punto de la flecha máxima

$Z_{\text{máx}} =$ flecha máxima

$\dot{E}(a) = \dot{E}(0) =$ pendiente (o ángulo) en el extremo izquierdo de la viga.

$\dot{E}(b) = \dot{E}(L) =$ pendiente (o ángulo) en el extremo derecho de la viga.

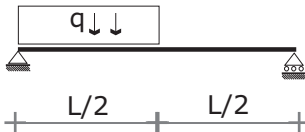


$$\dot{E}(x) = \frac{q}{24EI} (L^3 - 6Lx^2 + 4x^3)$$

$$Z(x) = \frac{qx}{384EI} (L^3 - 2Lx^2 + x^3)$$

$$\dot{E}(a) = \dot{E}(b) = \frac{qL^3}{24EI}$$

$$Z(c) = Z(\text{max}) = \frac{5qL^4}{384EI}$$



$$\dot{E}(x) = \frac{q}{384EI} (9L^3 - 72Lx^2 + 64x^3)$$

$$Z(x) = \frac{qx}{384EI} (9L^3 - 24Lx^2 + 16x^3)$$

$$0 \leq x \leq L/2$$

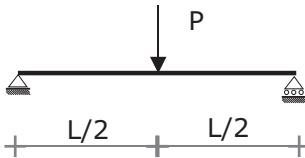
$$\dot{E}(a) = \frac{3qL^3}{128EI} \quad \dot{E}(b) = \frac{7qL^3}{384EI}$$

$$\dot{E}(x) = \frac{qL}{384EI} (24x^2 - 48Lx + 17L^2)$$

$$Z(x) = \frac{qL}{384EI} (8x^3 - 24Lx^2 + 17L^2x - L^3)$$

$$L/2 \leq x \leq L$$

$$Z(\text{max}) = \frac{5qL^4}{768EI}$$



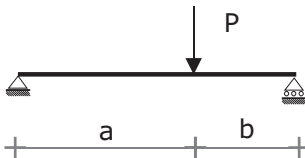
$$\dot{E}(x) = \frac{P}{16EI} (L^2 - 4x^2)$$

$$Z(x) = \frac{Px}{48EI} (3L^2 - 4Lx^2)$$

$$0 \leq x \leq L/2$$

$$\dot{E}(a) = \dot{E}(b) = \frac{PL^2}{16EI}$$

$$Z(c) = Z(\text{max}) = \frac{PL^3}{48EI}$$



$$\dot{E}(x) = \frac{Pb}{6LEI} (L^2 - b^2 - 3x^2)$$

$$Z(x) = \frac{Pbx}{6LEI} (L^2 - b^2 - x^2)$$

$$0 \leq x \leq a$$

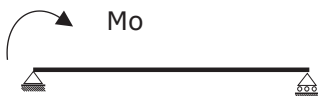
$$\dot{E}(a) = \frac{Pab(L+b)}{6LEI}$$

$$\dot{E}(b) = \frac{Pab(L+a)}{6LEI}$$

$$\text{Si } a \geq b, Z_c = \frac{Pb(3L^2 - 4b^2)}{48EI}$$

$$\text{Si } a \geq b, x_1 = \sqrt{\frac{L^2 - b^2}{3}}$$

$$Z(\text{max}) = \frac{Pb(L^2 - b^2)^{3/2}}{9\sqrt{3}LEI}$$



$$\dot{E}(x) = \frac{Mo}{6LEI} (2L^2 - 6Lx + 3x^2)$$

$$Z(x) = \frac{Mox}{6LEI} (2L^2 - 3Lx + x^2)$$

$$Z(c) = \frac{MoL^2}{16EI}$$

$$x_1 = L \left(\frac{1 - \sqrt{3}}{3} \right)$$

$$\dot{E}(a) = \frac{MoL}{3EI} \quad \dot{E}(b) = \frac{MoL}{6EI}$$

$$Z(\text{max}) = \frac{MoL^2}{9\sqrt{3}EI}$$